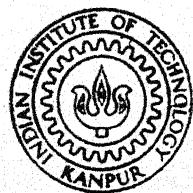


# OPTIMUM DESIGN OF MULTISTAGE VAPOUR-COMPRESSION REFRIGERATION SYSTEM

By

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DEPARTMENT OF MECHANICAL ENGINEERING  
INDIAN INSTITUTE OF TECHNOLOGY, KANPUR  
DECEMBER, 1983

# OPTIMUM DESIGN OF MULTISTAGE VAPOUR-COMPRESSION REFRIGERATION SYSTEM

A Thesis Submitted  
in Partial Fulfilment of the Requirements  
for the Degree of  
MASTER OF TECHNOLOGY

By

RAJARAM PRASAD

to the

DEPARTMENT OF MECHANICAL ENGINEERING  
INDIAN INSTITUTE OF TECHNOLOGY, KANPUR  
DECEMBER, 1983

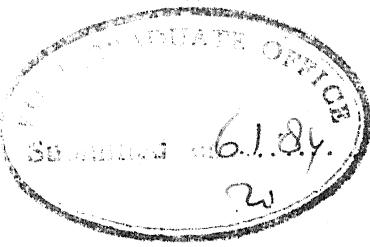
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CERTIFICATE

This is to certify that the thesis entitled  
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January, 1984.

ACKNOWLEDGEMENTS

I express my deep sense of gratitude to Drs. Manohar Prasad and Ashok Kumar Mittal for their valuable guidance and constant encouragement throughout the course of this work.

I gratefully acknowledge Messrs Frick India Ltd., New Delhi, Messrs United Refrigeration Works, Kanpur and Messrs Kalyan Cooling Corporation, Kanpur, for supplying costs of available sizes of various equipments. I also acknowledge Jal Sansthan, Kanpur and KESA House, Kanpur for supplying the costs of water and electricity for various years.

Sincere thanks are due to my friends Anil, Lalit and Nanduri for the valuable help rendered by them.

I am thankful to Mr. R.N. Srivastava for his efficient typing of the thesis.

- Rajaram Prasad

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NOMENCLATURE

COP	Coefficient of performance
$COP_C$	Coefficient of performance of carnot cycle
$c_{pw}$	Specific heat of water, kJ/Kg-°C
$c_{p1}, c_{p2}, c_{p4}$	Specific heat at constant pressure for the refrigerant vapour at the evaporator, the inter-stage and the condensing pressures, kJ/Kg-°C
$C_{comp}, C_{cond}$	Costs of compressor and condenser, Rs.
$c_e$	Cost rate of electricity, Rs/kWh
$c_w$	Cost rate of water, Rs/m <sup>3</sup>
$h_x$	Enthalpy of refrigerant at state x, kJ/Kg
L	Life of the system, years
$\dot{m}_i$	Mass flow rate of refrigerant through the compressor of ith stage, Kg/s
$\dot{m}_w$	Mass flow rate of cooling water in the condenser, Kg/s
n	Number of stages
p	Pressure, bar
$p_i$	Interstage pressure, bar
OF	Operating factor
R	Interest rate
$\dot{Q}_c$	Cooling capacity of the system, tons
$\dot{Q}_h$	Heat rejected in the condenser, kJ/s
r	Pressure ratio across the compressor
$\eta_v$	Volumetric efficiency
$HP_i$	Power required by the compressor of the ith stage, Metric Horse Power
$\eta_{ci}$	Adiabatic compression efficiency for the compressor of ith stage
T	Temperature, °C

ABSTRACT

A general computer program has been developed to optimize an n-stage vapour-compression refrigeration system based on minimum total cost comprising initial and running expenditures. The condensing temperature has also been treated as a decision variable and the costs of all the compressors and the condenser and the cost of cooling water have been included in the total cost. Optimum interstage pressure and condensing temperature have been determined for two-stage R-12 and R-22 systems, making use of available compressor and condenser sizes and their costs. Effects of subcooling, superheating, approach and compressor efficiency have been considered to make the results more relevant. Results have been presented for various design ambient temperatures and also considering year-round variations in ambient temperature. Optimum interstage pressures obtained by the present method have been compared with data available in the literature and has been found to be greater than that of all other investigators. It has been found to be about 15% higher than the geometric mean pressure.

## CHAPTER - 1

### INTRODUCTION

#### 1.1 DESCRIPTION:

It is well known that the Carnot refrigeration system having two reversible adiabatic and two reversible isothermal processes is the perfect system of refrigeration. Its coefficient of performance ( $COP_C$ ) depends entirely on the two temperature limits. But, in practice it is not possible to operate a Carnot cycle.<sup>(1)</sup> Yet it is desirable that any actual refrigeration system should have its coefficient of performance (COP) close to  $COP_C$ . It is sought to be achieved with the help of a phase change cycle consisting of an isothermal evaporation, a reversible adiabatic compression, an isobaric condensation having a very small non-isothermal process, and an isenthalpic expansion. The COP of this system tends to  $COP_C$ , if the difference between the condenser and the evaporator temperatures is small. But, in practice the condensing temperature is decided by the ambient temperature and the evaporator temperature is fixed by the cooling requirements. For low evaporator temperature, the single-stage vapour compression system referred to above has many disadvantages:

- (i) Increase in compression ratio reduces the volumetric efficiency of the compressor [ $\eta_v = 1 - (V_c/V_s) \cdot (p_2/p_1)^{1/n} - 1$  ]. It means that larger compressor displacement would be required for a specified capacity.

- (ii) Leakage past the piston is increased due to large pressure difference. Also there is overheating of the compressor body giving rise to lubrication problems, and the overall efficiency of the compressor is considerably reduced.
- (iii) Due to high pressure ratio across the expansion valve more flash vapour is produced after the expansion which reduces the refrigerating effect. On the other hand the work of compression is increased, reducing COP.
- (iv) Smaller the COP, larger is the heat rejection in the condenser [ $Q_{\text{rejected}} = Q_{\text{cooling}}(1 + 1/\text{COP})$ ]. This results in increased size of the condenser causing higher cost.

To overcome these difficulties, multistage (generally 2- or 3-stage) vapour compression system is generally used. In such a system the vapour from the evaporator is compressed to the condenser pressure in two or more stages with intercooling in-between. An n-stage system is exhibited in Figure 2.1 along with p-h diagram. A 2-stage system is shown in Figure 2.4. As compared to a single-stage system, the additional components in a 2-stage system are: a compressor, an expansion valve, and a flash intercooler. Liquid refrigerant together with some flashed vapour enters the evaporator (E), absorbs heat and gets converted into vapour. The vapour is then compressed by the LP compressor to an intermediate pressure (and not to the condensing pressure as is done in a single-stage system), at which the flash intercooler (FI) is maintained. The compressed vapour gets desuperheated by the liquid refrigerant in

the flash intercooler. The heat released due to desuperheating augments the vapour flow rate through the HP compressor. This vapour, the flash vapour due to throttling of the condensate and the LP saturated vapour are then compressed to the condenser pressure by the HP compressor where heat transfer to the cooling medium causes condensation of the vapour. The condensate is throttled upto flash intercooler pressure through the expansion valve (EV1). The saturated liquid from the flash intercooler is supplied to the evaporator through the expansion valve (EV2), completing the cycle.

Referring to Figures 2.3 and 2.5, it can be observed that the throttling process is closer to the saturated liquid line. It implies therefore that the refrigeration effect per unit mass is increased. On the other hand, on the compression side, the processes are closer to the saturated vapour line. It renders saving in work of compression and also less superheated vapour which means less heat rejection. Thus the limitations of a single stage-system are adequately overcome. Further, a multistage system also has some additional advantages:

- (i) By proper arrangement of the cranks for all the stages, torque can be made more even; thus requiring a lighter flywheel.
- (ii) Here LP compressor is subjected to lower pressure and so the cylinder wall thickness is smaller. Thus a saving in material cost occurs. Also, only the HP compressor needs to be made of special materials to withstand the high pressure.

But, in this system the number of auxiliaries and spare parts is more, giving rise to increased initial investment. However, this disadvantage is offset by reduced running costs for higher capacities and low temperature applications.

### 1.2 LITERATURE REVIEW:

The performance of a multistage system is dependent upon the interstage pressure(s) or temperature(s) selected. There exists a set of such pressure(s) which gives(give) the best performance of the system and it(they) is(are) called "Optimum interstage pressure(s)". It(they) is(are) obtained by maximizing or minimizing certain performance criterion (objective function) and this criterion must be explicitly spelt out. So far three optimization criteria have been envisaged. They are:

- (i) minimization of work input per Kg of refrigerant through the evaporator<sup>(1-5,7)</sup>
- (ii) maximization of the COP of the system<sup>(8)</sup>
- (iii) minimization of power per ton of cooling.<sup>(9)</sup>

The last two criteria are basically equivalent to each other and so give the same results. But the work input per Kg of refrigerant flow is not a complete indicator of the performance of the system because it does not include the variation in cooling produced.

In general for n-stage compression of perfect gas with perfect intercooling, the interstage pressures are in geometric progression given by

$$p_{i,1}/p_1 = p_{i,2}/p_{i,1} = \dots = p_n/p_{i,n-1} = (p_n/p_1)^{1/n}$$

for minimum work of compression and, in particular, for a two-stage compression the interstage pressure ( $p_i$ ) is given by

$$p_i = (p_h p_1)^{0.5} \quad (1.1b)$$

where  $p_h$  is the discharge pressure of the HP compressor and  $p_1$  is the suction pressure of the LP compressor. But  $p_i$  given above (eqn. (1.1b)) has been shown to be lower than that for maximum COP.<sup>(1)</sup>

Soumerai<sup>(2)</sup> has developed an iterative scheme which uses a graphical method to find the interstage pressure.

Keshwani and Rastogi<sup>(3)</sup> obtained the optimum interstage pressure corresponding to minimum work. They assumed the mass of the flashed vapour to be invariant with the intermediate pressure. This assumption is unrealistic. Also the pressure at which the mass of the flashed vapour evolves is not recommended. Surprisingly their results are found to be unrealistic for R-12 and R-22 systems.<sup>(16)</sup>

The work of Keshwani and Rastogi<sup>(3)</sup> has been extended by Ram Lal.<sup>(4)</sup> He accounted for the variation in mass of flashed vapour with intermediate pressure. But this method involves the generation of complete superheated properties which is quite cumbersome. Also it corresponds to minimum work. Verma and Charan<sup>(5)</sup> used linear variation in mass of flashed vapour with interstage pressure, rendering simplification in Ram Lal's work. Their expressions consist of a set of differential equations. But unfortunately any numerical result based on their analysis to check the method was not established. Their method is also based on minimum work.

Surprisingly the interstage pressure reported corresponds to maximum COP.

Gosney<sup>(6)</sup> has studied two-stage vapour — compression systems, but only for a few operating conditions. Arora and Dhar<sup>(7)</sup> have analysed n-stage system and reported results based on the minimum work criterion. Moreover, they have considered the ideal cycle without the effects of compression efficiency, subcooling and superheating. Such a cycle is an unrealistic representation of the actual system. Further, one has to resort to computer for calculation of optimum inter-stage pressure for each set of operating conditions.

Prasad<sup>(8)</sup> has developed a correlation for a two-stage R-12 system in terms of the condenser and evaporator temperatures. COP has been considered as the objective function and an approximate method, neglecting the superheat horn, has been used. It is very well valid for R-12 system because it has a very small area of superheat horn as compared to work of compression. The method completely avoids the use of superheated properties which is a great simplification without significant loss of accuracy. But it cannot, unfortunately, be applied to other systems because of a significant area of superheat horn as compared to the work of compression.

### 1.3 PRESENT STUDY:

In the past investigations, attention has been focussed on the thermodynamic performance of the system. In the present study the focus has been shifted to cost aspects of the system as a whole.

When we change from a single-stage system to a multi-stage system, the number of compressors, intercoolers and expansion valves increases. Optimization of the thermodynamic performance overlooks the costs of these components, and hence the calculation of optimum interstage pressure is more of an academic interest rather than that of commercial requirements. To make the result more relevant, the costs of these components has also been included. Electricity is consumed in running the compressors and cooling water is required for the condenser. So the costs of these items have been considered besides the costs of both the compressors.

Evidently for maximum COP the condensing temperature should be as low as possible and due to natural limitations it may be kept as close to the ambient temperature as possible. But as the condensing temperature is reduced, the mass flow rate of cooling water increases, giving rise to increased cost. For a two-stage system, we have thus both the intermediate temperature and the condensing temperature as decision variables leading to a 2-dimensional minimization problem. In the present work, rather than fixing the condensing temperature, an optimum condensing temperature has been obtained along with optimum cooling water rate and optimum interstage pressure with respect to minimum cost.

In the present study, effects of subcooling after condensation and superheating in the evaporator have been considered. Also the effect of variation in approach temperature (difference between condensing and outlet cooling water temperatures) on the optimum quantities has been considered.

The effects of tonnage capacity of the system has also been considered. Optimum quantities have been obtained for various evaporator temperatures. Also all the optimum quantities have been obtained for different design ambient conditions. Thus, for a known set of design parameters (viz. subcooling, super-heating, approach, the evaporator temperature and the tonnage) and for any given design ambient temperature of a given location, an optimum system can be recommended which includes size of the compressors for HP and LP side condenser size and economic cooling water rate.

As the compressors and other equipments are available only in certain discrete sizes, the cost function comes out to be a discontinuous function of the decision variables. A general computer program has been developed to minimize a function of  $n$  variables. Optimization has been done for two-stage R-12 and R-22 systems as the cost data for other refrigerant compressors were not received from the sources.

The work has been extended to include the effect of variations in ambient temperatures. In the absence of any control system, the condensing and the inter-stage temperature would change as the ambient temperature changes. Hence the system would deviate from optimality. But the prediction of these variations is not easy. In the present work, rather than optimizing for any one ambient temperature, a joint effect of monthly mean maximum temperatures for a particular place has been optimized. Kanpur has been selected as the place and the temperature data have been taken from reference (10).

For the present investigation, the costs of various components of the system have been obtained from Messrs Frick India Ltd., New Delhi, Messrs United Refrigeration Works, Kanpur and Messrs Kalyan Cooling Corporation, Kanpur. Cost of water has been obtained from Jal Sansthan, Kanpur. Cost of electricity was obtained for various years from KESA House, Kanpur and a logistic curve has been fitted through this past data to forecast costs for the future. Properties of refrigerants have been used from correlations obtained by Prasad. <sup>(13)</sup>

CHAPTER - 2PROBLEM FORMULATION

## 2.1 MATHEMATICAL MODELLING:

## 2.1.1 n-stage System:

Figure 2.1 shows a generalized system consisting of n-stage compression with flash intercooling. In this system the evaporator vapour is compressed through successive stages with desuperheating in the flash intercoolers until the vapour is discharged into the condenser. To derive generalized results, the  $i^{\text{th}}$  stage as shown in Figure 2.2 is considered. Mass and energy conservation for the flash intercooler of this stage yields:

$$\dot{m}_i = \dot{m}_{i-1} (h_{2i-2} - h_{4n-2i+3}) / (h_{2i} - h_{4n-2i+1}) \quad (2.1)$$

for  $i = 2, 3, \dots, n$

where  $\dot{m}_i$  is the mass flow rate through the  $i^{\text{th}}$ -stage compressor.

Also,

$$\dot{m}_1 = 3.5 \dot{Q}_c / (h_1 - h_{4n-1}) \quad (2.2)$$

Metric Horse Power required by the  $i^{\text{th}}$  compressor is:

$$HP_i = \dot{m}_i (h_{2i} - h_{2i-1}) / 0.736 \quad (2.3)$$

If  $\eta_{ci}$  be the compression efficiency of the  $i^{\text{th}}$  stage, one can get

$$h_{2i} = h_{2i-1} + (h_{2i} - h_{2i-1}) \eta_{ci} \quad (2.4)$$

where  $h_{2i}$  is the enthalpy at the end of isentropic compression.

For the entire system,

$$COP = 4.75543 \dot{Q}_c / \sum_{i=1}^n h_{p_i} \quad (2.5)$$

Heat rejected in the condenser is:

$$\dot{Q}_h = \dot{m}_n (h_{2n} - h_{2n+1}) \quad (2.6)$$

Also,

$$\dot{Q}_h = 3.5 \dot{Q}_c + 0.736 \sum_{i=1}^n h_{p_i} \quad (2.7)$$

If  $T_h$  be the temperature rise of the cooling water in the condenser, the mass flow rate of cooling water is:

$$\dot{m}_w = \dot{Q}_h / (c_{pw} \cdot \Delta T_w) \quad (2.8)$$

$\Delta T_w$  needed in this expression is calculated on certain assumptions as explained below:

Referring to the real condensation process shown in the T-s diagram in Figure 2.5, we observe that the condensation process consists of three parts:

- (i) desuperheating of vapour, process A-B (in actual practice it occurs in two parts — (a) desuperheating and (b) combined desuperheating and condensation)
- (ii) condensation of vapour to liquid, process B-C
- (iii) subcooling of the condensate, process C-D.

The total heat rejection is given by the area A-B-C-D-d-a-A, consisting of heat rejections in the above three processes. But it is extremely difficult to predict the zone near point B where both condensation and desuperheating continue having extremely

complicated heat transfer process. On the other hand, if we consider the entire condensation to be occurring at a fixed temperature  $T_h$ , the heat rejected will be given by the area E-F-d-a-E which is smaller than the original area by an amount given by the area AEB minus the area CFD. But this difference is not appreciable in comparison to the total area and if required it may be neglected in the light of a major simplification. Validity of this approximation is also supported by other researchers.<sup>(15)</sup> However, the computation has been carried out for the same heat rejection. Only the condensing temperature has not been adjusted due to expected negligible change in  $T_h$ . So with this approximation, referring to Figure 2.6, we see that the exit water temperature is given by

$$T_{wo} = T_h - T_{ap}$$

and so,

$$\Delta T_w = T_{wo} - T_{wi} = T_h - T_{wi} - T_{ap} \quad (2.9)$$

### 2.1.2 Two-stage System:

For a two-stage system (Figure 2.4), we have  $n = 2$  and  $i = 1$  and 2. For these values of  $n$  and  $i$  one gets from eqns.

(2.1)-(2.7),

$$\dot{m}_1 = 3.5 \dot{Q}_c / (h_1 - h_7) \quad (2.10a)$$

$$\dot{m}_2 = \dot{m}_1 (h_2 - h_7) / (h_4 - h_5) \quad (2.10b)$$

$$HP1 = \dot{m}_1 (h_2 - h_1) / 0.736 \quad (2.10c)$$

$$HP2 = \dot{m}_2 (h_4 - h_3) / 0.736 \quad (2.10d)$$

$$h_2 = h_1 + (h_{2'} - h_1)/\eta_{c1} \quad (2.10e)$$

$$h_4 = h_3 + (h_{4'} - h_3)/\eta_{c2} \quad (2.10f)$$

$$COP = 4.75543 \dot{Q}_c / (HP1 + HP2) \quad (2.10g)$$

$$\dot{Q}_h = 3.5 \dot{Q}_c + 0.736(HP1 + HP2) \quad (2.10h)$$

$\dot{m}_w$  is given by eqn. (2.8) itself.

Correlations are available for saturated properties and specific heats (for finding enthalpies of superheated states) as a function of saturation temperature.<sup>(13)</sup> Thus, we can find all the enthalpies:

$$h_1 = h_{1'} + c_{p1} \cdot T_s \quad (2.11)$$

$$s_1 = s_{1'} + c_{p1} \ln[1 + T_s/(T_1 + 273.15)] \quad (2.12)$$

$$h_{2'} = h_3 + c_{p2}(T_i + 273.15) [\exp \{ (s_1 - s_3)/c_{p2} \} - 1] \quad (2.13)$$

$$h_{4'} = h_{4''} + c_{p4}(T_h + 273.15) [\exp \{ (s_3 - s_{4''})/c_{p4} \} - 1] \quad (2.14)$$

$$h_5 = h_f + T_h - T_c \quad (2.15)$$

where,

$T_1$  = evaporator temperature

$T_h$  = condenser temperature

$T_s$  = superheating in the evaporator

$T_c$  = subcooling of the condensate

$T_i$  = interstage temperature.

$\eta_{c1}$  and  $\eta_{c2}$  are known functions of pressure ratios  $p_2/p_1$  and  $p_4/p_3$  respectively and have been taken from reference (12) as:

$$\begin{aligned} 1/\eta_c &= 1.13623 + 0.113289 r - 0.0334529 r^2 \\ &\quad + 0.486757 \times 10^{-2} r^3 - 0.2134 \times 10^{-3} r^4 \end{aligned} \quad (2.16)$$

where  $r$  is the pressure ratio. The pressures are themselves functions of saturation temperatures.<sup>(13)</sup> Thus all the above expressions given in eqns. (2.1)-(2.16) can be expressed in terms of temperatures. An explicit expression has not been attempted at as it becomes extremely involved. Moreover, it serves no useful purpose because we can directly use the above implicit expressions in computer program to evaluate the objective function.

Thus, we finally find the following functional relationships:

$$HP1 = HP1(T_i, T_h; T_l, T_{wi}, T_{ap}, T_s, T_c, \dot{Q}_c) \quad (2.17a)$$

$$HP2 = HP2(T_i, T_h; T_l, T_{wi}, T_{ap}, T_s, T_c, \dot{Q}_c) \quad (2.17b)$$

$$\dot{Q}_h = \dot{Q}_h(T_i, T_h; T_l, T_{wi}, T_{ap}, T_s, T_c, \dot{Q}_c) \quad (2.17c)$$

$$\dot{m}_w = \dot{m}_w(T_i, T_h; T_l, T_{wi}, T_{ap}, T_s, T_c, \dot{Q}_c) \quad (2.17d)$$

$$COP = COP(T_i, T_h; T_l, T_{wi}, T_{ap}, T_s, T_c, \dot{Q}_c) \quad (2.17e)$$

## 2.2 OBJECTIVE FUNCTION:

### 2.2.1 For Design Ambient Conditions:

In the present work the objective function considered is the average annual cost for the entire life span in terms of the present value. Thus the total cost incurred (in terms of the present value) over the entire life span of  $L$  years has been divided by  $L$  to compute the objective function. It consists of initial and running costs.

During any particular optimization, the evaporator size is fixed. So only the costs of all the compressors and the condenser has been included in the list of initial costs. The running costs include the cost of power (electricity), the cost of cooling water and the cost of maintenance.

The costs of the compressor and the condenser are functions of HP required for the compressor and heat rejection ( $\dot{Q}_h$ ) in the condenser respectively. Thus,

$$C_{\text{initial}} = \frac{1}{L} \left[ \sum_{i=1}^n C_{\text{comp}}^{(HP_i)} + C_{\text{cond}}^{(\dot{Q}_h)} \right] \quad (2.18)$$

For the cost of electricity a logistic curve of the form  $a/[b + \exp(-cx)]$  has been fitted through the past data and future costs have been forecast. Thus the cost rates of electricity (Rs/kWh)  $c_{e1}, c_{e2}, \dots, c_{eL}$  in the 1<sup>st</sup>, 2<sup>nd</sup>, ..., L<sup>th</sup> years can be obtained. Also the cost rate of the cooling water  $c_w$  (Rs/m<sup>3</sup>) has been obtained from "Jal Sansthan, Kanpur".

If R be the rate of interest, the present values of these costs can be computed as:

$$c_{e,\text{effective}} = \frac{1}{L} \sum_{i=1}^L c_{ei} / (1 + R)^{i-1} \quad (2.19a)$$

$$c_{w,\text{effective}} = \frac{1}{L} \sum_{i=1}^L c_w / (1 + R)^{i-1} \quad (2.19b)$$

So,

$$C_{\text{power}} = 6447.36 * \text{OF} * c_{e,\text{effective}} * \sum_{i=1}^n HP_i \quad (2.20)$$

and

$$C_{\text{water}} = 31536 * \text{OF} * c_{w,\text{effective}} * \dot{m}_w \quad (2.21)$$

in Rs/year, where OF is the part of the day for which the system has to operate and is called operating factor.

Maintenance cost has been considered as 10% of the equipment cost based on data given in reference (14). Thus,

$$C_{\text{maint.}} = 0.10 * C_{\text{initial}} * [(1 + R)^L - 1] / [R(1 + R)^{L-1}] \quad (2.22)$$

So,

$$C_{\text{running}} = C_{\text{power}} + C_{\text{water}} + C_{\text{maint.}} \quad (2.23)$$

The objective function (total annual cost) is then given by:

$$Z = C_{\text{initial}} + C_{\text{running}} \quad (2.24)$$

and its functional dependence is expressed by

$$Z = Z(T_i, T_h, T_l, T_{wi}, T_{ap}, T_s, T_c, Q_c, c_e, c_w, C_{\text{comp}}, C_{\text{cond}}) \quad (2.25)$$

Our aim will be to minimize this  $Z$  subject to the constraints:

$$T_l \leq T_i \leq T_h \quad (2.26)$$

and,

$$T_h > T_{wi} + T_{ap} \quad (2.27)$$

These relations express the fact that the interstage temperature must lie between the evaporator and the condensing temperatures and that the condensing temperature must exceed the inlet cooling water temperature plus the approach.

### 2.2.2 Variations in Ambient Temperatures:

In the previous section, the objective function has been formulated considering the ambient temperature as a

design parameter. Thus the cost can be minimized for any selected design ambient temperature. When there is no control mechanism to control the condensing and interstage temperatures at their optimal levels then the effect of variations in the ambient temperature should be incorporated in the analysis.

As the ambient temperature varies throughout the year, it is not easy to predict the deviations of  $T_h$  and  $T_i$  from optimal levels. In the present study, rather than optimizing the objective function for any single ambient temperature, a joint effect of the monthly temperature variations has been optimized. It has been still assumed that  $T_h$  and  $T_i$  are not varied when the ambient temperature changes. Thus if  $T_{w1}$ ,  $T_{w2}$ , ...,  $T_{w12}$  be the monthly average water temperatures for the months of January, February, ..., December, then the mass flow rates of cooling water corresponding to these temperatures will be

$$\dot{m}_{wj} = \dot{Q}_h / \{c_{pw}(T_h - T_{wj} - T_{ap})\} \quad (2.28)$$

We consider an average of all these water flow rates and take

$$\dot{m}_{w,av} = \frac{\dot{Q}_h}{12c_{pw}} \sum_{j=1}^{12} \left( \frac{1}{T_h - T_{wj} - T_{ap}} \right) \quad (2.29)$$

We use this  $\dot{m}_{w,av}$  to compute the cost of water. This procedure amounts to a kind of randomization over the ambient temperatures. Now we have

$$C_{water} = 31536 * OF * c_{w,effective} * \dot{m}_{w,av} \quad (2.30)$$

and  $C_{\text{initial}}$ ,  $C_{\text{power}}$ ,  $C_{\text{maint.}}$  are still given by the expressions (2.18), (2.20), (2.22), respectively. Also the final objective function  $Z$  is given by the same expression (2.24) and its functional dependence now becomes:

$$Z = Z(T_i, T_h, T_1, T_{\text{ap}}, T_s, T_c, \dot{Q}_c, c_e, c_w, C_{\text{comp}}, C_{\text{cond}}) \quad (2.31)$$

Thus this new modification only changes the cost of water. With this approach we do not get an optimum design for any particular ambient conditions, but the total cost for the year as a whole will be minimized.

CHAPTER - 3OPTIMIZATION

## 3.1 NATURE OF THE FUNCTION:

Our aim is to minimize the cost of the system expressed by eqns. (2.25) and (2.31). The first step in a systematic approach to the solution of the problem requires a knowledge of the nature of the function and accordingly the technique to be adopted is to be selected.

The objective function  $Z$  is a function of two variables  $T_i$  and  $T_h$  for a given set. Variations in  $Z$  with either of the variables, keeping the other variable fixed, are shown in Figures 3.1a and 3.1b for typical cases. The jumps appearing in these plots correspond to a change in one of the compressor capacities as compressors are available only in certain discrete sizes. Whenever a bigger or a smaller size compressor is required, there is an increase or decrease in the initial cost component. This gives rise to sudden jump in the objective function. As the interstage temperature  $T_i$  is increased from a lower value, power for the LP compressor increases and the power for the HP compressor decreases. Hence a change in the HP compressor corresponds to a downward jump while a change in the LP compressor corresponds to an upward jump.

Thus the objective function is discontinuous at various points where the size of a compressor changes. At these points the function is not differentiable and hence the

usual method of optimizations involving derivatives of the function cannot be used. For such a case Powell's search technique<sup>(17)</sup> is used being an ideal choice.

### 3.2 OPTIMIZATION TECHNIQUE:

Figure 3.2a shows a flowchart for minimizing a general function  $f(x_1, x_2, \dots, x_n)$  of  $n$  variables:  $x_1, x_2, \dots, x_n$ . Considering  $\vec{X} = (x_1, x_2, \dots, x_n)$ , an  $n$ -dimensional vector, the function can also be expressed as  $f(\vec{X})$ . The method uses a sequential search technique. In any cycle of minimization, a set of  $n$  directions  $\vec{D}_1, \vec{D}_2, \dots, \vec{D}_n$  are needed, and to start with these are considered as rows of an  $n \times n$  identity matrix. Minimization is first done along each of these  $n$  directions, as explained later on. Having done that a new pattern direction  $\vec{D}_{n+1}$  is generated by subtracting the initial starting point  $\vec{X}_0$  from the finally reached point  $\vec{X}_n$  (after minimizations along each of the  $n$  directions). At this point the flowchart shows a logical unit to decide whether to retain the old set of directions or change them. Thus if either  $f_3 \geq f_1$  or  $\delta \geq 0$  (see the flowchart), the old set of directions  $(\vec{D}_1, \vec{D}_2, \dots, \vec{D}_n)$  are retained and again minimizations are done along each of these directions, unless convergence has already been achieved. To the contrary, if both  $f_3 < f_1$  and  $\delta < 0$  then the old set of directions are changed as follows: the direction  $\vec{D}_m$  along which there had been maximum decrease in the function value is now replaced by  $\vec{D}_{n+1}$ , the new pattern direction. Having done this now again minimizations are done along each of this new set of directions, unless

convergence has already been reached. This completes one cycle of minimizations, and this cycle is repeated again and again until the final points and corresponding function values stop differing by a predetermined small quantity. The convergence is then said to have been achieved and the process is terminated, thus getting the minimum of the function  $f(\vec{X})$ .

Figure 3.2b shows a flowchart for minimizing  $f(\vec{X})$  in a particular direction  $\vec{D}$ , starting from a point  $\vec{X}$ . This problem is equivalent to finding a  $\alpha = \alpha^*$  so as to minimize  $f(\vec{X} + \alpha \vec{D})$ . For any given  $\vec{X}$  and  $\vec{D}$  one can write,

$$f(\vec{X} + \alpha \vec{D}) = F(\alpha), \text{ a function of } \alpha.$$

So the problem is basically a one dimensional minimization of  $F(\alpha)$ . Quadratic interpolation technique<sup>(18)</sup> has been used for this purpose. The technique can be well understood by the flowchart. In this method a quadratic curve (parabola) is fitted through three points of which at least one lies on either side of the minimum. Thus if  $a, b, c$  be three such points (values of  $\alpha$ ), then a quadratic through these points can be taken as

$$G(\alpha) = a_1 + a_2 \alpha + a_3 \alpha^2 \quad (3.1)$$

where the constants  $a_1, a_2, a_3$  are to be determined from the simultaneous equations

$$a_1 + a_2 a + a_3 a^2 = F(a) = z_a \quad (3.2a)$$

$$a_1 + a_2 b + a_3 b^2 = F(b) = z_b \quad (3.2b)$$

$$a_1 + a_2 c + a_3 c^2 = F(c) = z_c \quad (3.2c)$$

The expressions for  $a_1$ ,  $a_2$ ,  $a_3$  in terms of  $a$ ,  $b$ ,  $c$ ,  $z_a$ ,  $z_b$  and  $z_c$  are written in the flowchart (Figure 3.2b). Now  $G(\alpha)$  is considered to be an approximation for  $F(\alpha)$  and the minimum of  $G(\alpha)$  lies at

$$\alpha^* = -a_1/(2a_3) \quad (3.3)$$

provided  $a_3 > 0$ , which is automatically ensured if at least one of  $a$ ,  $b$ ,  $c$  lies on either side of the minimum point. Now  $\alpha^*$  is considered as a candidate for the minimum of  $F(\alpha)$ . If the values of  $F(\alpha^*)$  and  $G(\alpha^*)$  differ by more than a predetermined small quantity then one of the three points  $a$ ,  $b$ ,  $c$  is replaced by  $\alpha^*$  and a new quadratic is fitted through the new set of points. The cycle is thus repeated till the convergence is achieved.

Flowchart in Figure 3.2a makes repeated reference to flowchart in Figure 3.2b for one dimensional searches. Thus the technique essentially consists of a series of one dimensional minimizations.

### 3.2.1 Cost Data:

For minimizing the objective function for any given set of conditions by the technique mentioned in Section 3.2, it needs to be evaluated again and again. For evaluating the objective function, costs of compressors, condensers, electricity and water are needed. The results presented in the following chapter are based on the costs of compressors and condensers obtained from Messers Eriek India Ltd. New Delhi

and tabulated in Appendix A. Cost of water used is that for commercial purposes as obtained from Jal Sansthan, Kanpur and it is also given in Appendix A. Cost of electricity for various years have been obtained from KESA House, Kanpur and are tabulated in Appendix A. A logistic curve of the form  $a/[b + \exp(-cx)]$  has been fitted through these past data as detailed in Appendix B. The choice of such a curve has been motivated by the yearly variations in the cost of electricity as shown in Figure 3.3. The property values of the refrigerants taken from reference (13) are given in Appendix C.

### 3.2.2 Termination Criterion:

The termination criteria are based on the differences between the temperatures and objective function values in successive cycles of minimizations. A change in the inter-stage or the condensing temperature of  $0.01^{\circ}\text{C}$  or less will have no significant effect on any of the quantities of interest. Also the annual costs for a 10 ton system are found to be in tens of thousands of rupees. So an amount of Rs. 10/- can be considered as insignificant in comparison to the total annual costs. Thus the termination criteria used during the minimization of the cost are as follows: if (i) the temperatures in successive cycles of minimizations differ by less than  $0.01^{\circ}\text{C}$ , and (ii) the objective function values in successive cycles of minimizations differ by less than 10 (Rs/yr) then the process be terminated, thus obtaining the optimum point. The corresponding optimal quantities can then be calculated using the expressions in Chapter 2. To ensure that the minimum

reached is the global minimum various starting points were tried. In cases involving more than one minimum, the one having minimum objective function value was taken as the global minimum.

### 3.3 TWO STAGE OPTIMIZATION PROCEDURE:

In the method outlined in the previous section, the minimum of the objective function is reached in a single step. However, keeping the discontinuities in the function in view, one has to try several starting points selected in the entire range. A major computational simplification would result if the discontinuities were removed, taking care of them separately. This can be achieved by adopting a two-stage technique. The first stage involves extensive computer simulation. The entire  $T_i$ ,  $T_h$  plane is divided into a number of grids and performance of the system is studied for each of these grids (a set of operating conditions). This study helps determine the points where the sizes of the compressors are to be changed and the range of temperatures in which each of the combination of compressors works. Also, writing down the objective function values in the corresponding grids in the  $T_i$ ,  $T_h$  plane, a zone of low cost range can be identified as explained in Figure 3.4. This zone is the area enclosed by the bold lines in that figure.

The second stage involves finer optimization in the grids included in the low cost zone. As the combination of compressor sizes which work in each of the grids has already been determined in the first step, the cost of these compressors can be used for optimizations in the corresponding grids.

R-12  
 $Q_C = 10 \text{ tons}$   
 $T_C = 0 = T_s$   
 $T_{ap} = 3^\circ\text{C}$   
 $T_{wi} = 30^\circ\text{C}$

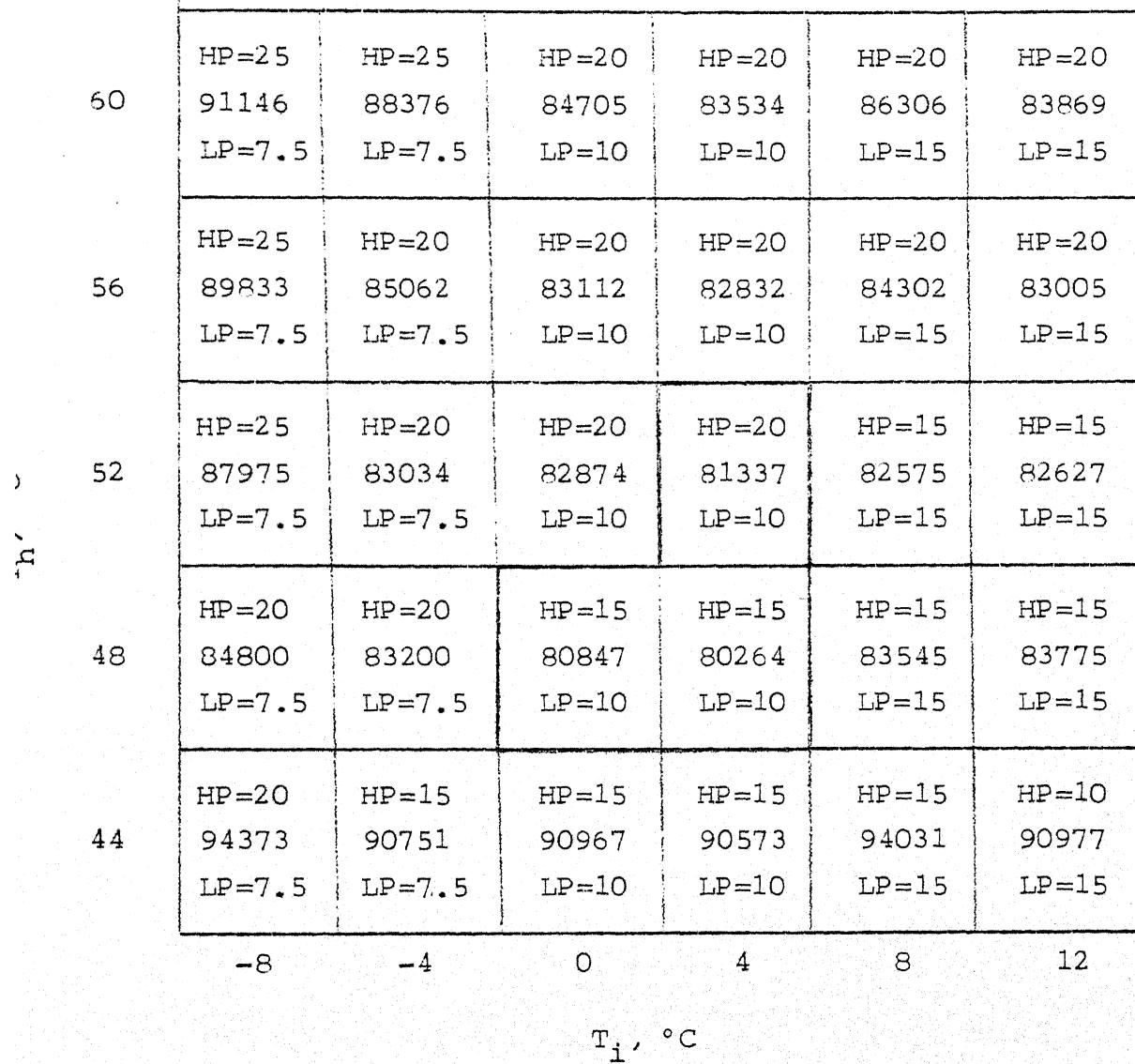


FIG. 3.4 Determination of Low Cost Zones. Numbers in the Grids are Costs (Rs/yr) and the Sizes of the Two Compressors.

Thus the costs of compressors remain fixed during optimization in each grid, and the discontinuous element of the objective function is removed.

The costs of the objective function in each of the grids are shown in Figure 3.4 together with the combinations of LP and HP compressors that would work in each of those grids, for a typical case. Three grids have been identified there as lying in a low cost zone. But in two of them the same combination of compressors works. So finally, only two combinations of compressors remain to be tried. Optimization has been done with starting points in each of the grids and the result has been found to correspond to that obtained by the earlier method. The present method, though more time consuming, gives a better insight into the problem.

CHAPTER - 4RESULTS AND DISCUSSIONS

The objective function formulated in Chapter 2 has been optimized for various ambient conditions, approach temperatures, degrees of subcooling, degrees of superheat and for various tonnage capacities of the system. The results being presented in the following sections pertain to a two-stage system for refrigerants R-12 and R-22. Life of the system has been taken as 15 years and an operating factor of 0.75 has been assumed. In Section 4.5 results are presented for the case when the variations in ambient conditions are incorporated as discussed in Section 2.2.2. Results of the present study have also been compared with available results in the literature.

#### 4.1 GRAPHICAL PRESENTATION OF RESULTS:

Figure 4.1a shows the variation in optimum interstage pressure for a 10 ton R-12 system with the evaporator temperature ( $T_1$ ) for various inlet water temperatures ( $T_{wi}$ ). Inlet water temperatures are kept same as ambient temperatures. In this plot an approach temperature of 3°C has been considered with no subcooling or superheating. Figure 4.1b shows a similar variation for R-22 system. From these figures it is possible to read out the value of  $P_{i\text{opt}}$  for any evaporator temperature between -50°C to -10°C and ambient temperature between 15°C to 35°C, for the refrigerants R-12 and R-22. As the evaporator temperature increases, the interstage pressure

increases for optimum sharing between the two stages. Also as the ambient temperature is increased mass flow of cooling water will tend to increase, thus necessitating an upward adjustment in the condensing temperature. So for the optimum sharing between the two stages  $P_{i\text{opt}}$  increases with increased ambient temperature. Variations for both R-12 and R-22 are similar, the only difference being that the interstage pressures are greater for R-22 than for R-12 for any given case.

Figures 4.2a and 4.2b show similar variations in optimum interstage temperatures for R-12 and R-22 systems, respectively. Whereas in case of interstage pressure the rate of increase with increasing evaporator temperature goes on increasing, it shows a reverse trend with the interstage temperature. This is due to the relationship between the saturation temperature and saturation pressure for these refrigerants.

Figures 4.3a and 4.3b show the variations in the condensing temperature with evaporator temperature having the ambient temperature as parameter. As the evaporator temperature is increased the optimum condensing temperature is also increased but the increase is not very pronounced. On the other hand the condensing temperature is more sensitive to the ambient temperature. It is so because the mass flow rate of the cooling water is directly dependent on the difference between the condensing temperature and the ambient temperature. Thus as the ambient temperature increases, the condensing temperature is also to be increased for reducing the cooling

water flow and the corresponding cost. The nature of variation for both R-12 and R-22 systems are similar.

Figures 4.4a and 4.4b show the variations in optimum cooling water rate with evaporator and ambient temperatures, keeping all other parameters fixed. As there is negligible variations in optimum operating conditions with respect to capacity of the system, the cooling water rate is directly proportional to the capacity of the system. For any ambient and evaporator temperatures one can read the value of optimum cooling water rate from these graphs. This flow rate should be maintained for maintaining the condensing and interstage temperatures at the optimum levels. Hence it is a very important quantity. It is evident from the figures that as the evaporator temperature is increased the mass flow of cooling water decreases. It is so because as the evaporator temperature is raised, work of compression reduces so that there is less heat rejection required. When the ambient temperature reduces, the water flow required also decreases. It is so because in that case the temperature rise of water increases so that the same heat is carried by a smaller quantity of cooling water.

Power requirements for LP and HP compressors are shown in Figures 4.5a and 4.5b for R-12 and R-22 systems respectively. As the evaporator temperature is raised, the power requirements for both LP and HP compressors decrease. The fall is more steep for the HP compressor. The power requirement for the HP compressor is seen to be greater than that for the LP compressor in all the cases. Further, the

difference between the powers required for HP and LP compressors is more for lower evaporator temperatures and tends to become zero as the evaporator temperature increases beyond  $-10^{\circ}$

Values of COP for optimum operating conditions are shown in Figures 4.6a and 4.6b for refrigerants R-12 and R-22 respectively. It is seen that for corresponding evaporator and ambient temperatures, R-12 has a slightly higher COP than R-22 when the interstage and condensing temperatures are maintained at their optimum levels.

#### 4.2 EFFECTS OF VARIOUS PARAMETERS:

##### 4.2.1 Effect of Subcooling and Superheating:

Effect of subcooling on optimum values of interstage pressure, condensing temperature and cooling water rate are shown in Figures 4.7a, 4.7b and 4.7c, respectively. It is seen that as subcooling of condensate is increased, the optimum interstage pressure comes down. Referring to eqn. (2.10b), it is seen that as  $h_5$  is reduced due to subcooling  $\dot{m}_2$  (the mass flow through HP compressor) gets reduced, which means less load on HP compressor. To balance it  $P_{i\text{opt}}$  gets lowered down.

The optimum condensing temperature increases as the subcooling is increased. This is also due to the same reason as mentioned above, i.e. the condensing temperature is slightly increased to balance the reduced load on the HP compressor. Also, as the condensing temperature is increased, temperature rise of the cooling water is more and so the mass flow rate of cooling water should get reduced with subcooling. This is well depicted by Figure 4.7c.

Table 4.1 shows the effect of superheating of vapour in evaporator on optimum interstage and condensing temperature for a typical situation. It is seen that with increasing degree of superheat both optimum interstage pressure and condensing temperature increase slightly but the effect is negligible.

For any given subcooling and superheating, the optimum quantities ( $P_{i\text{opt}}$ ,  $T_{h\text{opt}}$ ,  $\dot{m}_{w\text{opt}}$ ,  $\text{COP}_{\text{opt}}$ ) can be determined by using the following multipliers:

$$F_{Pi} = \frac{P_{i\text{opt}} \left( T_1, T_{wi}, T_{ap}, T_c, T_s \right)}{P_{i\text{opt}} \left( T_1, T_{wi}, T_{ap}, 0, 0 \right)} \quad (4.1a)$$

$$F_{Th} = \frac{T_{h\text{opt}} \left( T_1, T_{wi}, T_{ap}, T_c, T_s \right) + 273.15}{T_{h\text{opt}} \left( T_1, T_{wi}, T_{ap}, 0, 0 \right) + 273.15} \quad (4.1b)$$

$$F_{mw} = \frac{\dot{m}_{w\text{opt}} \left( T_1, T_{wi}, T_{ap}, T_c, T_s \right)}{\dot{m}_{w\text{opt}} \left( T_1, T_{wi}, T_{ap}, 0, 0 \right)} \quad (4.1c)$$

and

$$F_{\text{COP}} = \frac{\text{COP}_{\text{opt}} \left( T_1, T_{wi}, T_{ap}, T_c, T_s \right)}{\text{COP}_{\text{opt}} \left( T_1, T_{wi}, T_{ap}, 0, 0 \right)} \quad (4.1d)$$

The following expressions have been obtained for R-12 by regression analysis:

$$F_{Pi} = 1 - 0.01654 T_c - 0.00661 T_c^2 + 0.000105 T_c^3 \quad (4.2)$$

$$F_{Th} = 1 + 0.000256 T_c + 0.000175 T_c^2 - 0.0000125 T_c^3 \quad (4.3)$$

$$F_{mw} = 1 - 0.01701 T_c + 0.003257 T_c^2 - 0.000210 T_c^3 \quad (4.4)$$

Analogous expressions for the case of R-22 system are:

$$F_{Pi} = 1 - 0.1010 T_c - 0.00377 T_c^2 + 0.000104 T_c^3 \quad (4.6)$$

$$F_{Th} = 1 + 0.000188 T_c + 0.000106 T_c^2 - 0.0000117 T_c^3 \quad (4.7)$$

$$F_{mw} = 1 - 0.01569 T_c + 0.00251 T_c^2 - 0.000157 T_c^3 \quad (4.8)$$

$$F_{COP} = 1 + 0.01001 T_c - 0.00211 T_c^2 + 0.000169 T_c^3 \quad (4.9)$$

The effect of superheating has been neglected. Thus, first the optimum quantities are to be found from the graphs given in Figures 4.1 to 4.6 for the case without subcooling and superheating, and then these quantities are to be multiplied by the corresponding multipliers given above.

#### 4.2.2 Effect of Approach Temperatures:

In the above sections all the results have been presented for 3°C approach. But, in individual cases they may be required to be different than 3°C. So the effect of various approach temperatures on the optimum quantities have been considered. It has been found that the optimum values of interstage pressure, condensing temperature and cooling water rate all increase as the approach temperature is increased. But optimum value of COP reduces with increasing approach temperatures. However, the effect of approach temperature is more pronounced on cooling water rate than the other quantities. It is so because the temperature rise of the cooling water is directly related to the approach temperature. Figure 4.8 quantitatively shows the effect of approach on the cooling water rate.

For approach other than 3°C, multipliers defined by:

$$G_{Pi} = \frac{P_{i\text{opt}}(T_1, T_{wi}, T_{ap}, T_c, T_s)}{P'_{i\text{opt}}(T_1, T_{wi}, 3, T_c, T_s)} \quad (4.10a)$$

$$G_{Th} = \frac{T_{h\text{opt}}(T_1, T_{wi}, T_{ap}, T_c, T_s) + 273.15}{T'_{h\text{opt}}(T_1, T_{wi}, 3, T_c, T_s) + 273.15} \quad (4.10b)$$

$$G_{mw} = \frac{\dot{m}_{w\text{opt}}(T_1, T_{wi}, T_{ap}, T_c, T_s)}{\dot{m}'_{w\text{opt}}(T_1, T_{wi}, 3, T_c, T_s)} \quad (4.10c)$$

$$G_{COP} = \frac{COP_{\text{opt}}(T_1, T_{wi}, T_{ap}, T_c, T_s)}{COP'_{\text{opt}}(T_1, T_{wi}, 3, T_c, T_s)} \quad (4.10d)$$

have been determined by regression analysis. For R-12 system, these are:

$$G_{Pi} = 0.96830 - 0.0001363 T_{ap} + 0.002106 T_{ap}^2 \quad (4.11)$$

$$G_{Th} = 0.95736 - 0.0005398 T_{ap} + 0.002380 T_{ap}^2 \quad (4.12)$$

$$G_{mw} = 0.80860 + 0.09529 T_{ap} - 0.00938 T_{ap}^2 \quad (4.13)$$

$$G_{COP} = 1.02809 - 0.001064 T_{ap} - 0.001938 T_{ap}^2 \quad (4.14)$$

and for R-22 system,

$$G_{Pi} = 0.99144 - 0.0025634 T_{ap} + 0.001407 T_{ap}^2 \quad (4.15)$$

$$G_{Th} = 0.98296 - 0.0033177 T_{ap} + 0.002476 T_{ap}^2 \quad (4.16)$$

$$G_{mw} = 0.78609 + 0.10942 T_{ap} - 0.012483 T_{ap}^2 \quad (4.17)$$

$$G_{COP} = 1.01943 + 0.00340 T_{ap} - 0.002300 T_{ap}^2 \quad (4.18)$$

Thus to compute the various quantities for approach temperature different than 3°C, one has to multiply the results with 3°C approach by the corresponding multipliers given above.

#### 4.3 ILLUSTRATION:

It is to select the optimum 2-stage R-12 vapour-compression refrigeration system for a given set of requirements and to recommend the optimum operating conditions.

Available Information:

1. Capacity of the plant: 10 tons
2. Evaporator temperature required: -35°C
3. Ambient temperature: 30°C
4. Approach between the condensing and exit cooling water temperatures: 5°C
5. Degree of subcooling of the condensate possible: 5°C.

Solution:

First the results are found for the case without subcooling and superheating and with approach temperature of 3°C.

From Figure 4.1a,

Optimum interstage pressure,  $P'_{i\text{opt}} = 3.5 \text{ bar}$

From Figure 4.2a,  $T'_{i\text{opt}} = 4.2^\circ\text{C}$

From Figure 4.3a, Optimum condensing temperature,  $T'_{h\text{opt}} = 48.5^\circ\text{C}$

From Figure 4.4a, optimum cooling water rate,  $\dot{m}'_{w\text{opt}} = 0.853 \text{ Kg/s}$

From Figure 4.5a,

Power required by LP-compressor = 12.3 HP, and

Power required by HP-compressor = 15.5 HP.

So both LP and HP compressors need to be of 15 HP capacity, since the capacity next higher to 10 HP is 15 HP.

From Figure 4.6a,

COP of the system for optimum conditions = 1.735

To correct for approach temperature, subcooling and superheating the multipliers given by eqns. (4.2-4.5 and 4.11-4.14) are computed as:

$$F_{Pi} = 1 - 0.01654 \times 5 - 0.00661 \times 5^2 + 0.000105 \times 5^3 \\ = 0.76517$$

$$F_{Th} = 1.00409, \quad F_{mw} = 0.97012, \quad F_{COP} = 1.032375$$

$$G_{Pi} = 0.96830 - 0.0001363 \times 5 + 0.002106 \times 5^2 = 1.02027$$

$$G_{Th} = 1.01416, \quad G_{mw} = 1.05055, \quad G_{COP} = 0.97432$$

So the corrected values of various quantities are:

$$P_{i\text{opt}} = P_{i\text{opt}}' \times F_{Pi} \times G_{Pi} = 3.5 \times 0.76517 \times 1.02027$$

i.e.,

$$P_{i\text{opt}} = 2.630 \text{ bar}$$

$$T_{h\text{opt}} = (T_{h\text{opt}}' + 273.15) \times F_{Th} \times G_{Th} - 273.15$$

(Note: The multipliers for the condensing temperature are obtained in terms of absolute temperatures.)

$$\text{So, } T_{h\text{opt}} = 54.33^\circ\text{C}, \quad \dot{m}_{w\text{opt}} = 0.86934 \text{ Kg/s}$$

$$\text{COP}_{\text{opt}} = 1.745$$

Thus, one selects:

Interstage pressure = 2.630 bar  
 Condensing temperature = 54.38°C  
 Cooling water rate = 0.86934 Kg/s  
 Capacity of LP-compressor = 15 HP  
 Capacity of HP-compressor = 15 HP.

#### 4.4 COMPARISON WITH AVAILABLE DATA:

In the present study, both the condensing and the interstage temperature have been considered as decision variables and their optimum values have been found together to minimize the total annual cost of the system. But the results available in literature have all been obtained for fixed condensing temperature with only interstage pressure as the decision variable. Thus to ensure a meaningful comparison the condensing temperature has been fixed and one dimensional minimization has been carried out with respect to interstage temperature. The results, thus obtained, are shown in Table 4.2, together with results available in the literature. It is seen that the optimum interstage pressure is about 9% more for  $T_h = 42.9^\circ\text{C}$  and 15% more for  $T_h = 55.5^\circ\text{C}$  as compared with the geometric mean pressure. The difference between the present study and other investigators is seen to be small for  $T_h = 42.9^\circ\text{C}$ . But, for  $T_h = 55.5^\circ\text{C}$ , the difference is significant. It is 8% more as compared to the result of Verma et al<sup>(5)</sup> and 4.5% more as compared to that of Prasad et al.<sup>(19)</sup>

The optimum interstage pressure by the present method is seen to be greater than that of all other investigators.

It is mainly due to the shift in the optimization criterion.

#### 4.5 VARIATIONS IN AMBIENT TEMPERATURE:

The effect of year-round variations in the ambient temperature has been incorporated as discussed in Section 2.2. Figures 4.9a, 4.9b, 4.9c and 4.9d show respectively the variations of optimum values of condensing temperature, interstage pressure, power requirements of LP and HP compressors and cooling water rate, with the evaporator temperature. The nature of their variations is similar to those for the case when the ambient temperature is considered to be a fixed design value. There is a difference between the two cases only in numerical values as can be seen by comparing the graphs for the two cases. Table 4.3 shows a comparison between the two cases for a 10 ton 2-stage R-12 system. The maximum ambient temperature has been considered as the design ambient temperature for the first case. It is seen that when the effect of the variations in ambient temperature is considered, both the cooling water rate and the annual cost comes down. Thus, for the evaporator temperature of  $-30^{\circ}\text{C}$ ,  $-40^{\circ}\text{C}$  and  $-50^{\circ}\text{C}$ , savings of Rs.1982/-, Rs.6928/- and Rs.8917/- result every year.

CHAPTER - 5CONCLUSIONS AND SUGGESTIONS

## 5.1 CONCLUSIONS:

From the present study the following conclusions are arrived at:

1. There is a significant difference between optimal systems obtained from thermodynamic and cost considerations. The total cost of the system comprising initial and running expenditures for the entire life (in terms of present value) has been considered as the criterion of optimization.
2. The optimum interstage pressures found in the present study are greater than those found by all other investigators. The present value is found to be about 15% higher than the geometric mean pressure.
3. The condensing temperature has also been considered as a decision variable because the cost of cooling water is directly related to it. An optimum condensing temperature, together with economic cooling water rate has been found out.
4. Multipliers have been determined to account for the effects of subcooling, superheating and various approach temperatures, within insignificant deviations. The optimum interstage pressure decreases with subcooling and slightly increases with superheating in the evaporator. On the other hand, the optimum condensing

temperature increases while the economic cooling water rate reduces as the subcooling is increased. The variation in approach temperature has maximum effect on economic cooling water rate.

5. The power requirement for HP-compressor has been found to be more than that for the LP-compressor. The difference between these power requirements is more for lower evaporator temperatures and tends to zero as the evaporator temperature is increased beyond  $-10^{\circ}\text{C}$ .
6. The system has been optimized both for a fixed design ambient temperature and incorporating the effect of year-round variations in the ambient temperature. The system optimized for the latter case has lesser total cost as compared to that for the first case. Thus, even though the system is not optimal for any particular ambient temperature, it is optimal for the year as a whole.
7. The expressions and methodology developed are general and can be used for any set of ambient conditions and prevailing costs. Also the method can be used for any refrigerant for which the correlations for the properties are available.

#### 5.2 SUGGESTIONS:

1. The best system would be the one which is optimal for all ambient temperatures round the year. But the components of the system has to be selected to meet the the worst ambient temperature. However, as the ambient temperature varies, the condensing temperature and the

interstage temperature will change. So a control device would be required to maintain the optimum interstage temperature for each ambient temperature.

2. In the present study the cooling load has been considered to be fixed. But as the ambient temperature varies the load also varies. The present work can be extended to include the effect of variations in the cooling load with time.

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APPENDIX - A

## A.1 Cost of Compressors for Refrigerant R-12:

Capacity (HP)	0.5	1.0	2.0	3.0	5.0	7.5	10.0	15.0	20
Cost (Rs.)	1650	2050	3260	4850	8270	17800	26400	55900	7580

## A.2 Cost of Compressors for R-22:

Capacity (HP)	3	5	7.5	10	15	20
Cost (Rs.)	4800	6500	7500	18000	25000	45000

## A.3 Cost of Water for Kanpur City:

Year	For House Use	For Industrial Use
1970-1978	Rs. 0.25/m <sup>3</sup>	Rs. 0.35/m <sup>3</sup>
1979-1982	Rs. 0.65/m <sup>3</sup>	Rs. 1.65/m <sup>3</sup>

## A.4 Cost of Electricity:

Year	1972	1976	1977	1978	1979	1981	1983
Cost (Rs/kWh)	0.36	0.43	0.45	0.50	0.55	0.60	0.65

APPENDIX - CPROPERTIES OF REFRIGERANTS

## C.1 Refrigerant R-12:

$$c_p(T) = 0.59524 + 0.0018175 (T + 15)$$

$$s_f(T) = 0.142093 + 0.33747 T/100 - 0.0399606(T/100)^2 + 0.019869(T/100)^3 + 0.014023(T/100)^4$$

$$h_g(T) = 188.86 + 0.440278 T - 7.02007(T/100)^2 - 5.07651(T/100)^3 - 3.82545(T/100)^4$$

$$h_f(T) = 36.1554 + 0.928108 T + 6.89916(T/100)^2 + 3.73414(T/100)^3 + 5.91673(T/100)^4$$

$$p(T) = 3.08968 + 0.10162 T + 12.6413(T/100)^2 + 6.74792(T/100)^3 + 1.01057(T/100)^4$$

## C.2 Refrigerant R-22:

$$c_p(T) = 0.60 + 0.0005246(T + 45 + 0.043686(T + 45)^2)$$

for -60 T 10

$$= 0.70114 + 0.0029529(T - 10 + 0.010472(T - 10)^2)$$

for 10 T 50

$$s_f(T) = 0.181132 + 0.43667 T/100 - 0.0596756(T/100)^2 - 0.0234343(T/100)^3 + 0.0638047(T/100)^4$$

$$h_g(T) = 251.106 + 35.4577(T/100) - 19.4993(T/100)^2 - 7.62005(T/100)^3 - 11.6756(T/100)^4$$

$$h_f(T) = 46.2102 + 1.20394 T + 6.828(T/100)^2 - 8.99387(T/100)^3 + 20.0612(T/100)^4$$

$$p(T) = 4.98105 + 0.151746 T + 19.5655(T/100)^2 + 9.8434(T/100)^3 + 2.22194(T/100)^4.$$

TABLE 4.1 Effect of Superheating on  $P_{i\text{opt}}$  and  $T_{h\text{opt}}$

$T_s$ , °C	$P_{i\text{opt}}$ , bar	$T_{h\text{opt}}$ , °C	
0	3.303	53.07	Refrigerant: R-12
5	3.308	53.08	$T_{wi} = 35^\circ\text{C}$
10	3.310	53.08	$T_c = 0^\circ\text{C}$
15	3.322	53.09	$T_{ap} = 3^\circ\text{C}$
20	3.328	53.10	$T_1 = -40^\circ\text{C}$

TABLE 4.2 Comparison of Optimum Interstage Pressures from Different Sources

Operating Conditions (°C)		Present Method	Ref.(19)	Ref.(5)	Ref.(7)	Ref.(12)	$(P_h P_1)$
$T_1$	$T_h$	bar	bar	bar	bar	bar	bar
-11.7	42.9	5.043	5.02	5.10	-	5.06	4.62
-11.7	55.5	6.112	5.85	-	5.66	5.99	5.34

TABLE 4.3 Comparison Between Results for Design Ambient Temperature and with Variations in Ambient Temperature

Operating Variables				With Effect of Variations in Ambient Temperature		With Design Conditions $T_{w\text{max}} = 40^\circ\text{C}$	
$\dot{Q}_c$ , tons	$T_1$ , °C	$T_c$ , °C	$T_{ap}$ , °C	$\dot{m}_{w\text{opt}}$ , Kg/s	$Z_{\text{opt}}$ , Rs/yr	$\dot{m}_{w\text{opt}}$ , Kg/s	$Z_{\text{opt}}$ , Rs/yr
10	-30	0	3	0.7901	85323	0.8821	87305
10	-40	0	3	0.8928	101713	0.9835	108641
10	-50	0	3	1.0492	123242	1.2053	132159

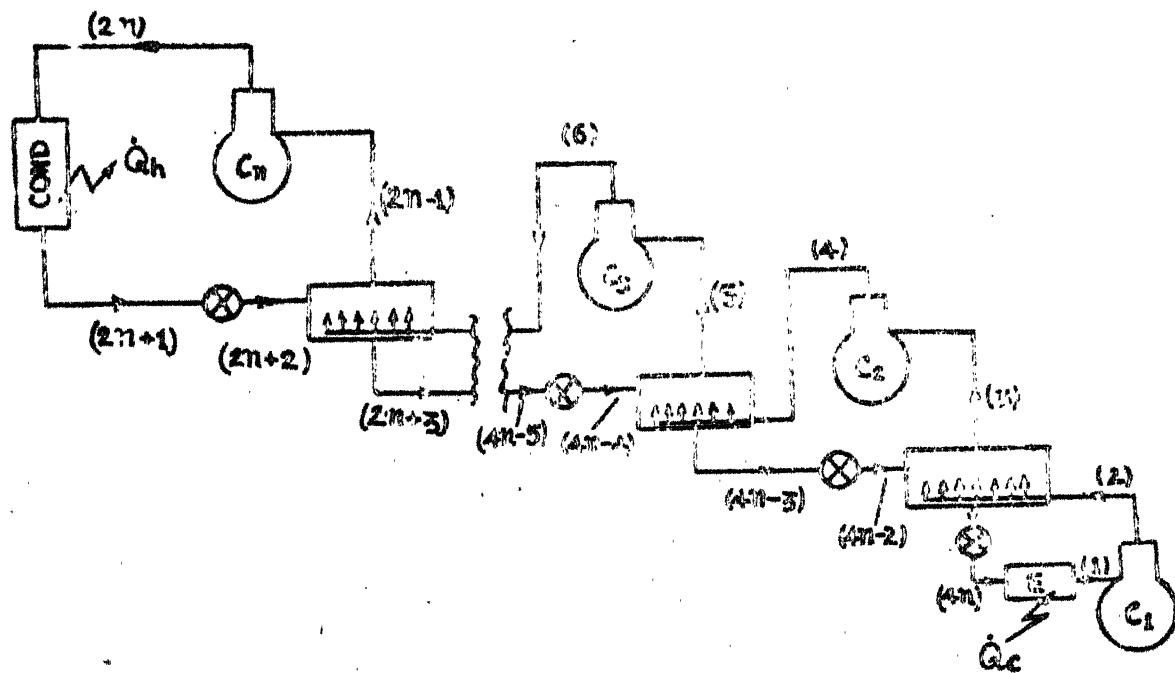


FIG. 2.1 SCHEMATIC DIAGRAM OF  
n-STAGE SYSTEM

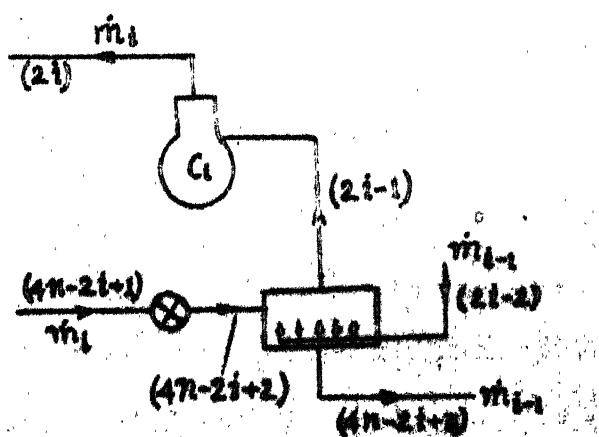


FIG. 2.2 1<sup>st</sup> STAGE OF AN  $n$ -STAGE SYSTEM

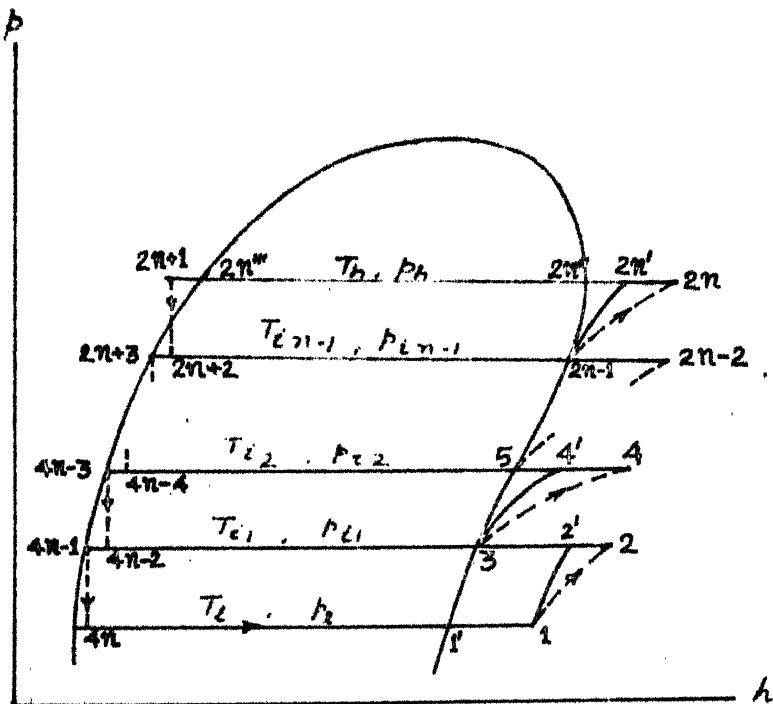


FIG. 2.3  $p$ - $h$  DIAGRAM FOR  $n$ -STAGE SYSTEM

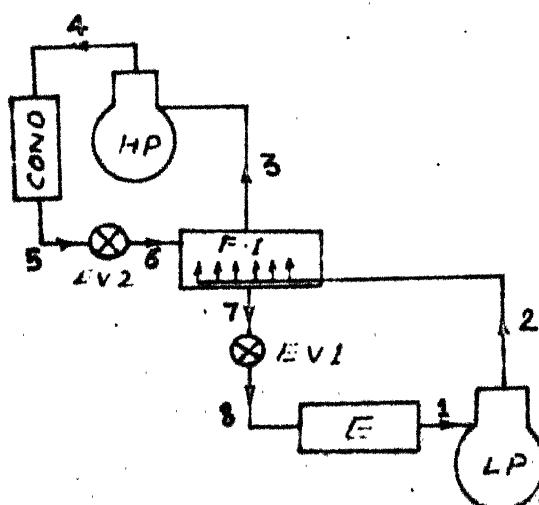


FIG 2.4 SCHEMATIC  
DIAGRAM OF 2-STAGE SYSTEM

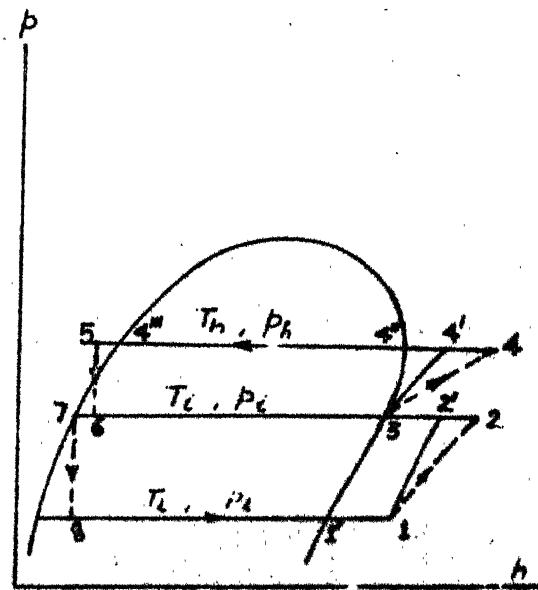


FIG. 2.5  $p$ - $h$  DIAGRAM  
OF 2-STAGE SYSTEM

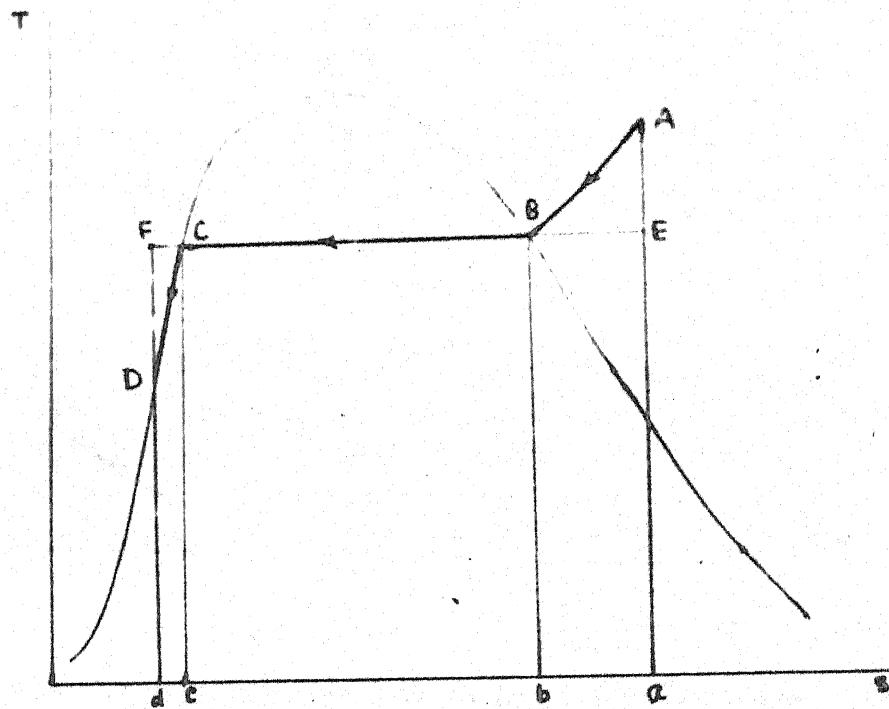


FIG. 2.6 A REAL CONDENSATION PROCESS  
ON T-S DIAGRAM

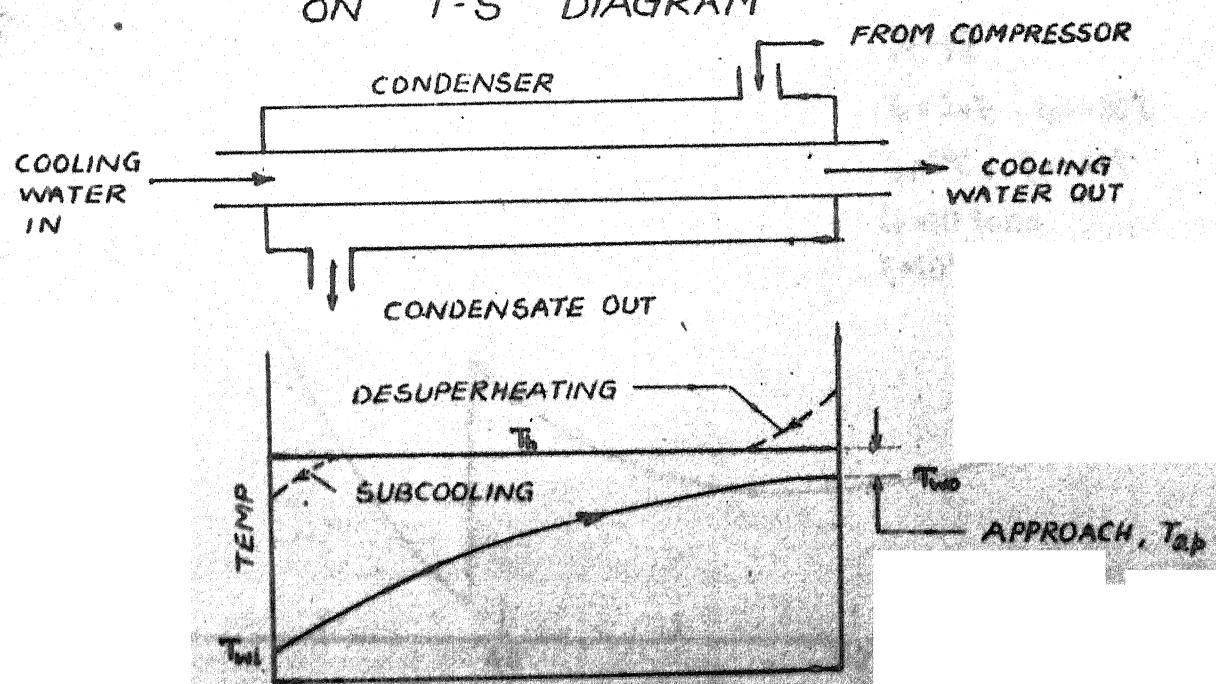


FIG. 2.7 SCHEMATIC DIAGRAM OF CONDENSATION  
SYSTEM AND TEMPERATURE VARIATIONS

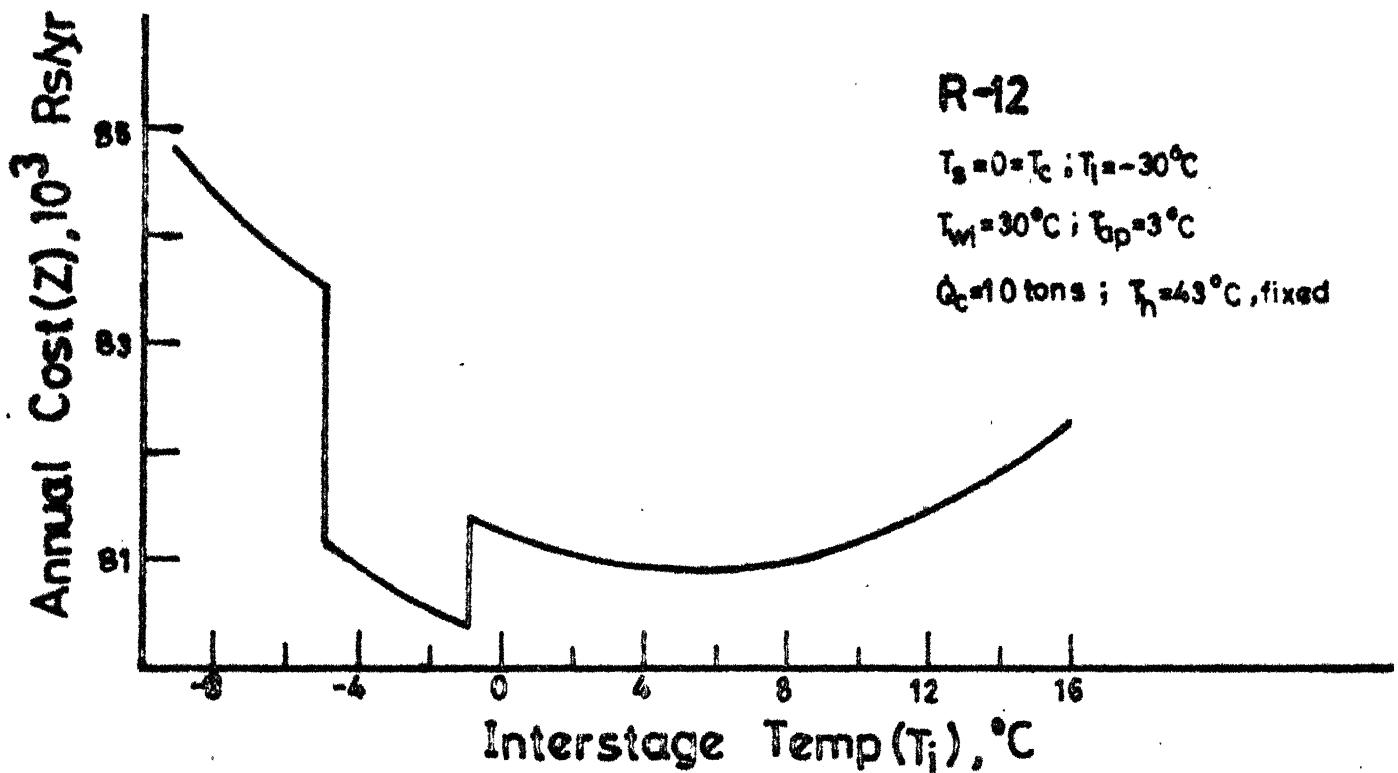


Fig 3-1a Variation of Annual Cost with Interstage Temp at a Fixed Condensing Temp

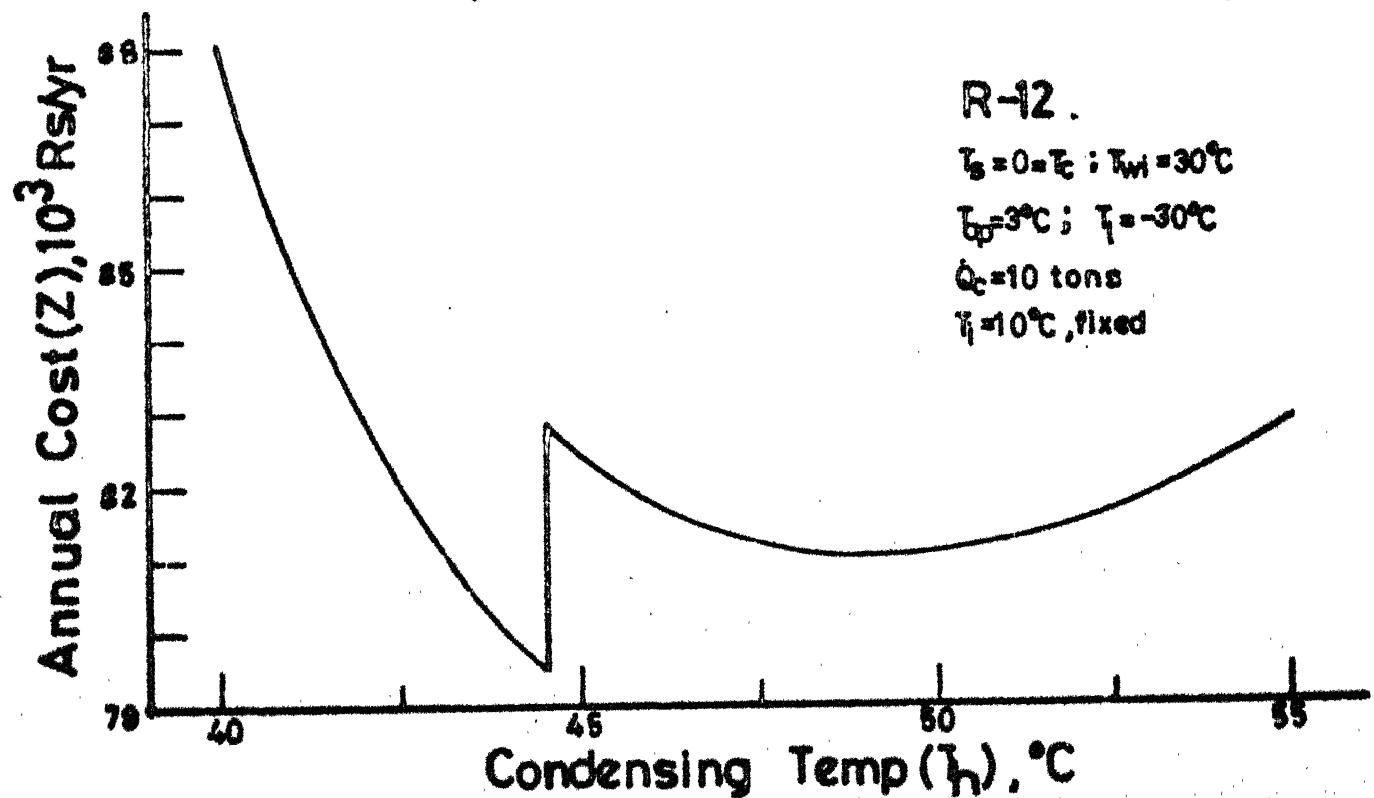


Fig 3-1b Variation of Annual Cost with Condensing Temp at a Fixed Interstage Temp

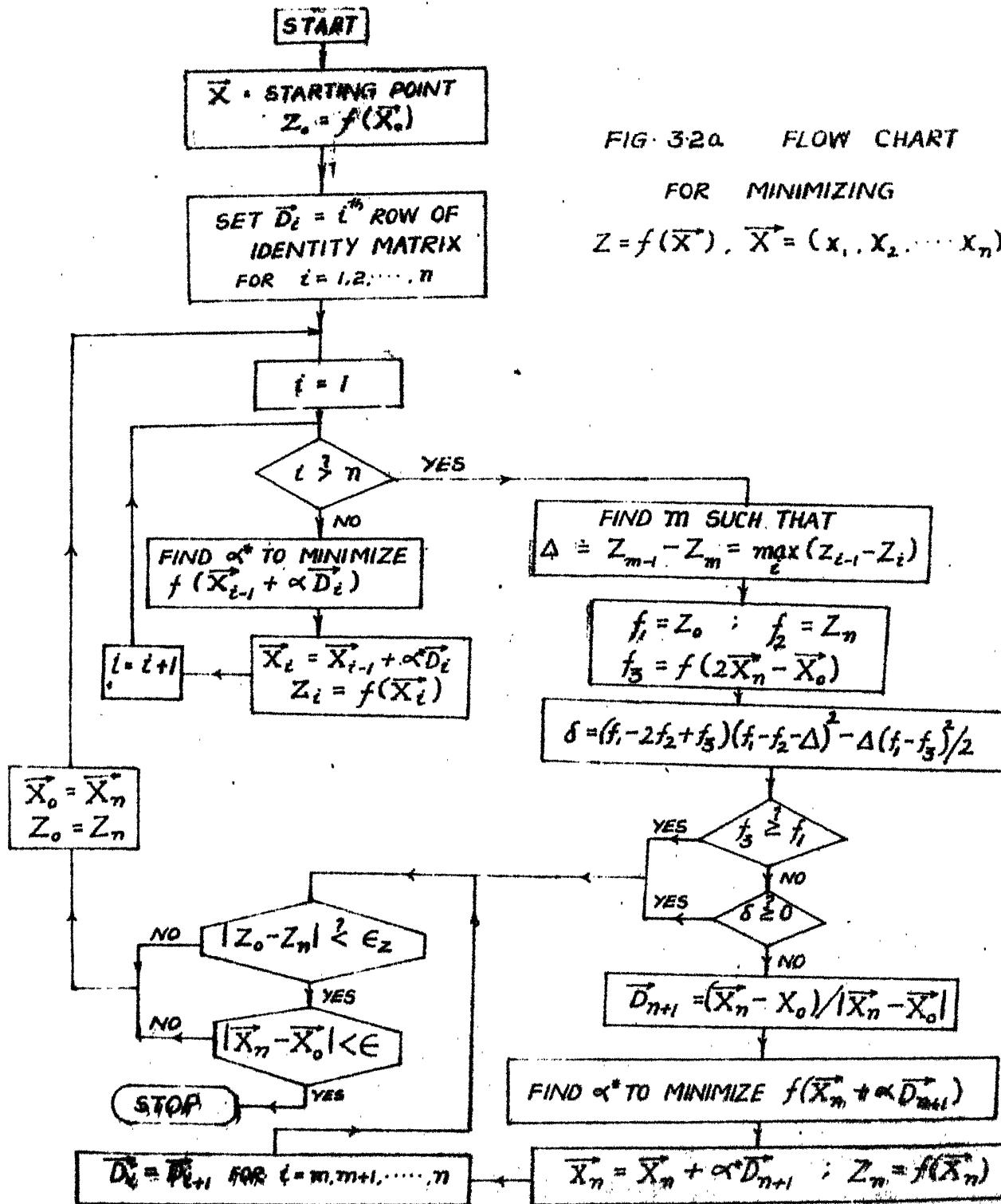


FIG. 3.2a. FLOW CHART

FOR MINIMIZING

$$Z = f(\vec{X}), \vec{X} = (x_1, x_2, \dots, x_n)$$

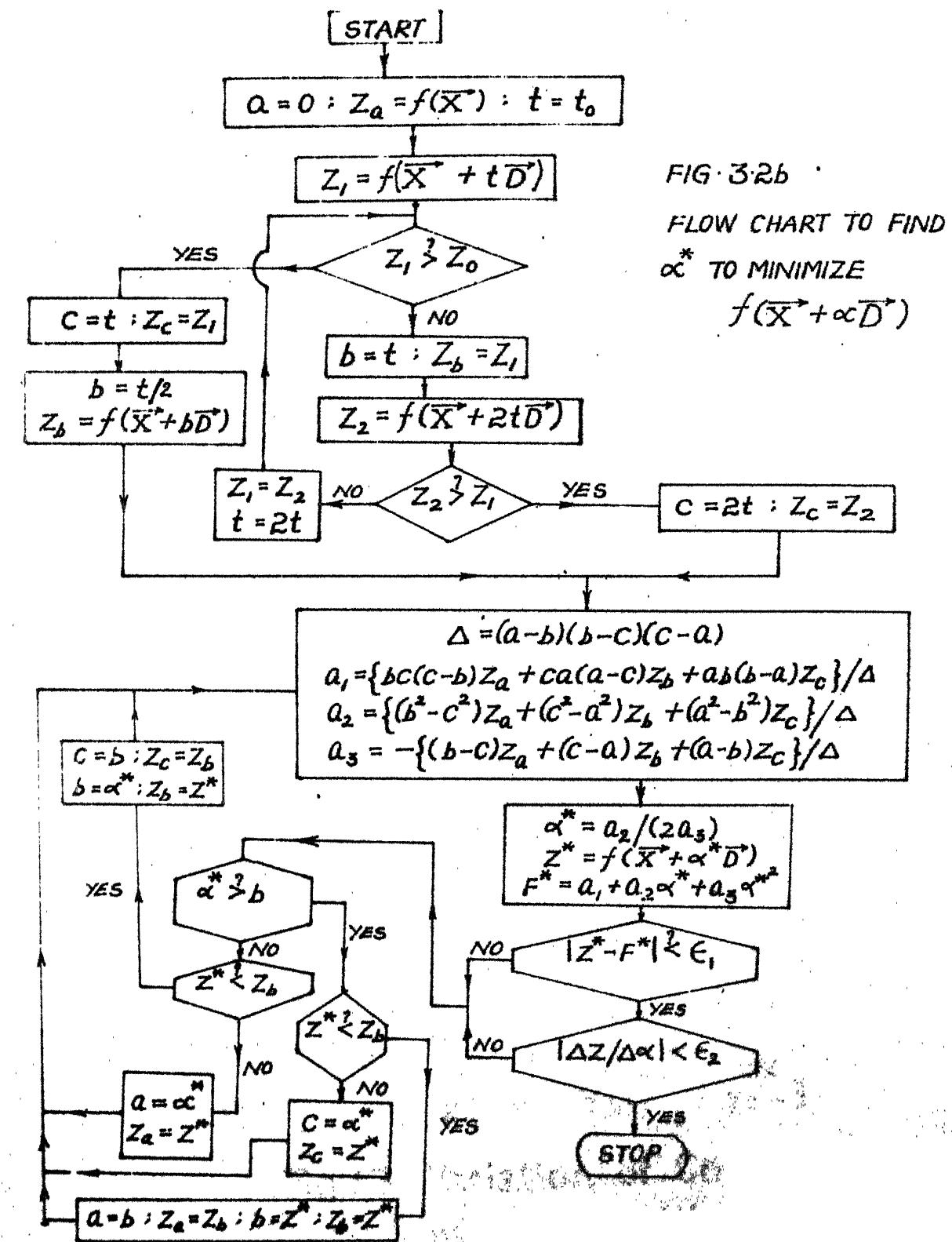


FIG. 3.2b  
FLOW CHART TO FIND  
 $\alpha^*$  TO MINIMIZE  
 $f(\vec{X} + \alpha\vec{D})$

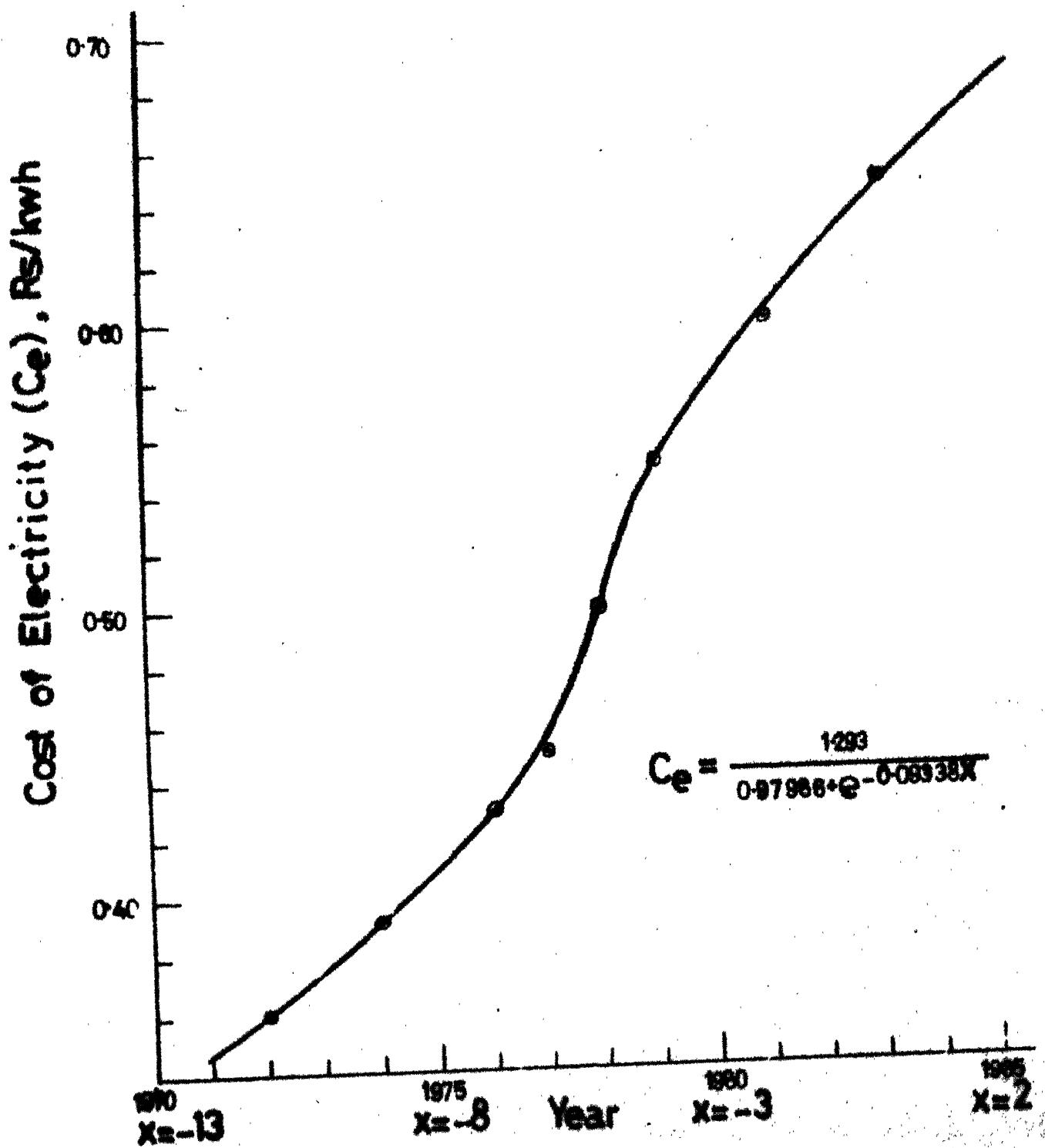


Fig 3.3 Variation of Cost of Electricity with Time (Year)

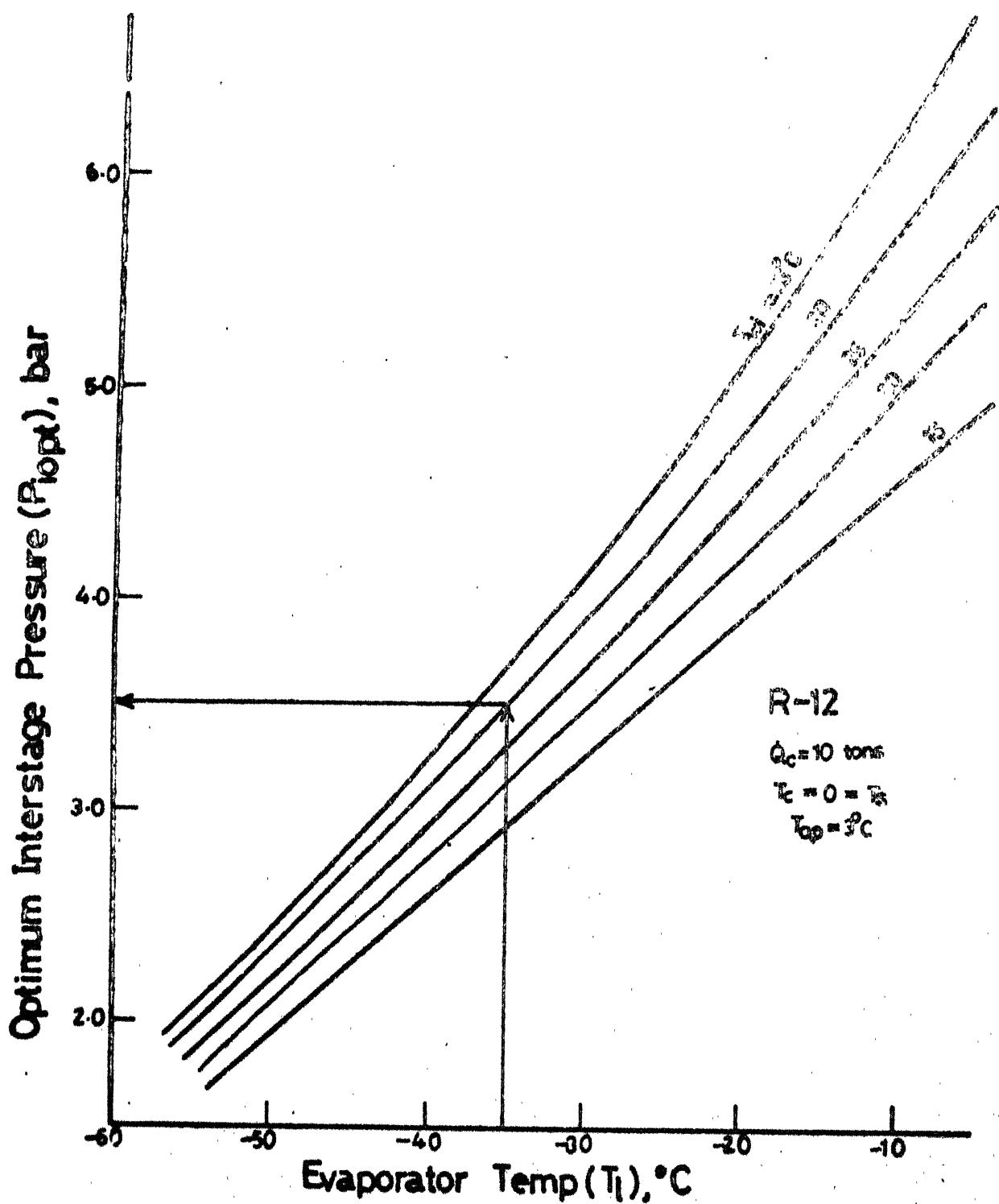
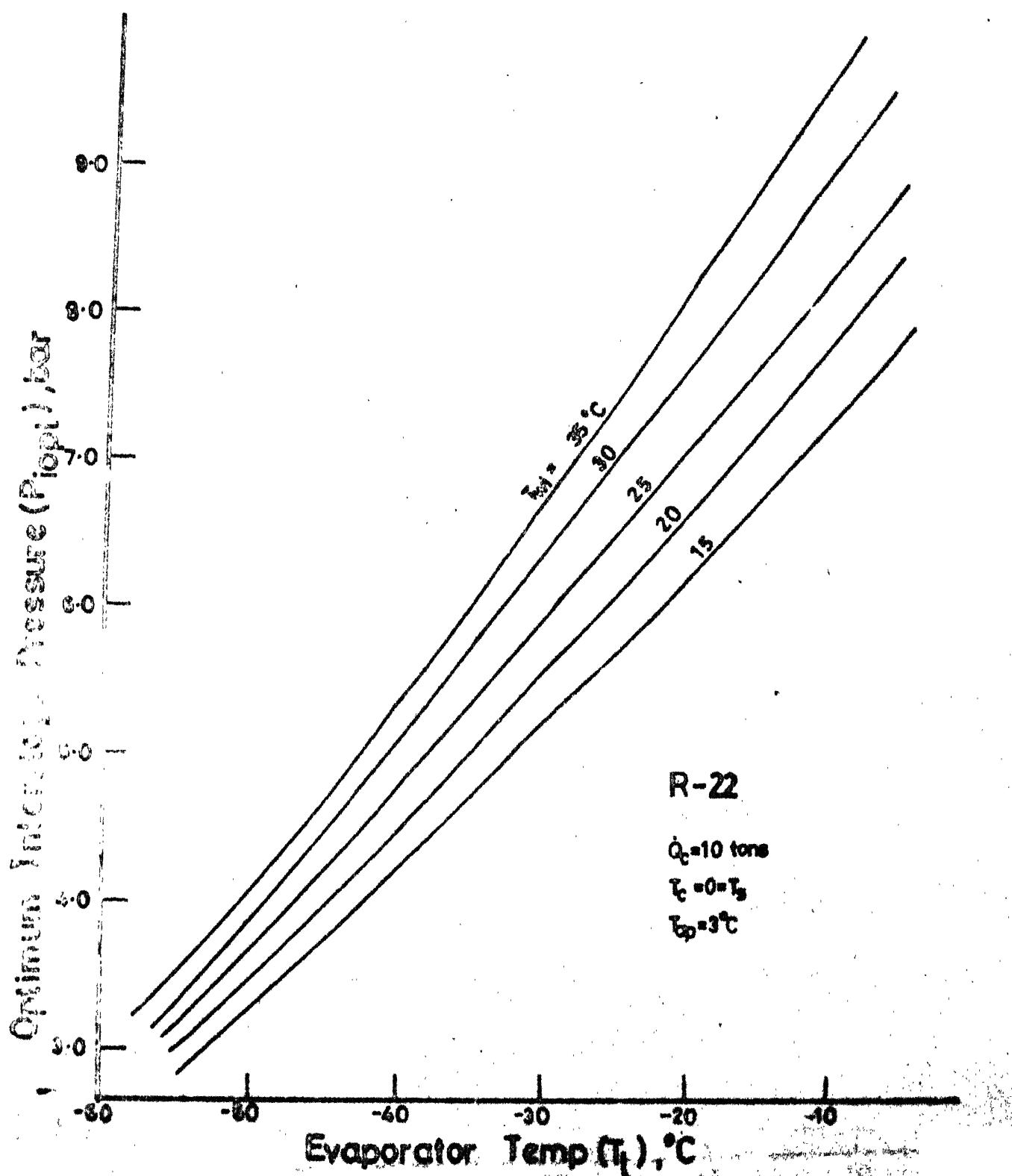


Fig 4.1a Variation of Optimum Interstage Pressure with Evaporator and Ambient Temps



**Fig 4.1b Variation of Interstage Pressure with Evaporator and Ambient Temperatures**

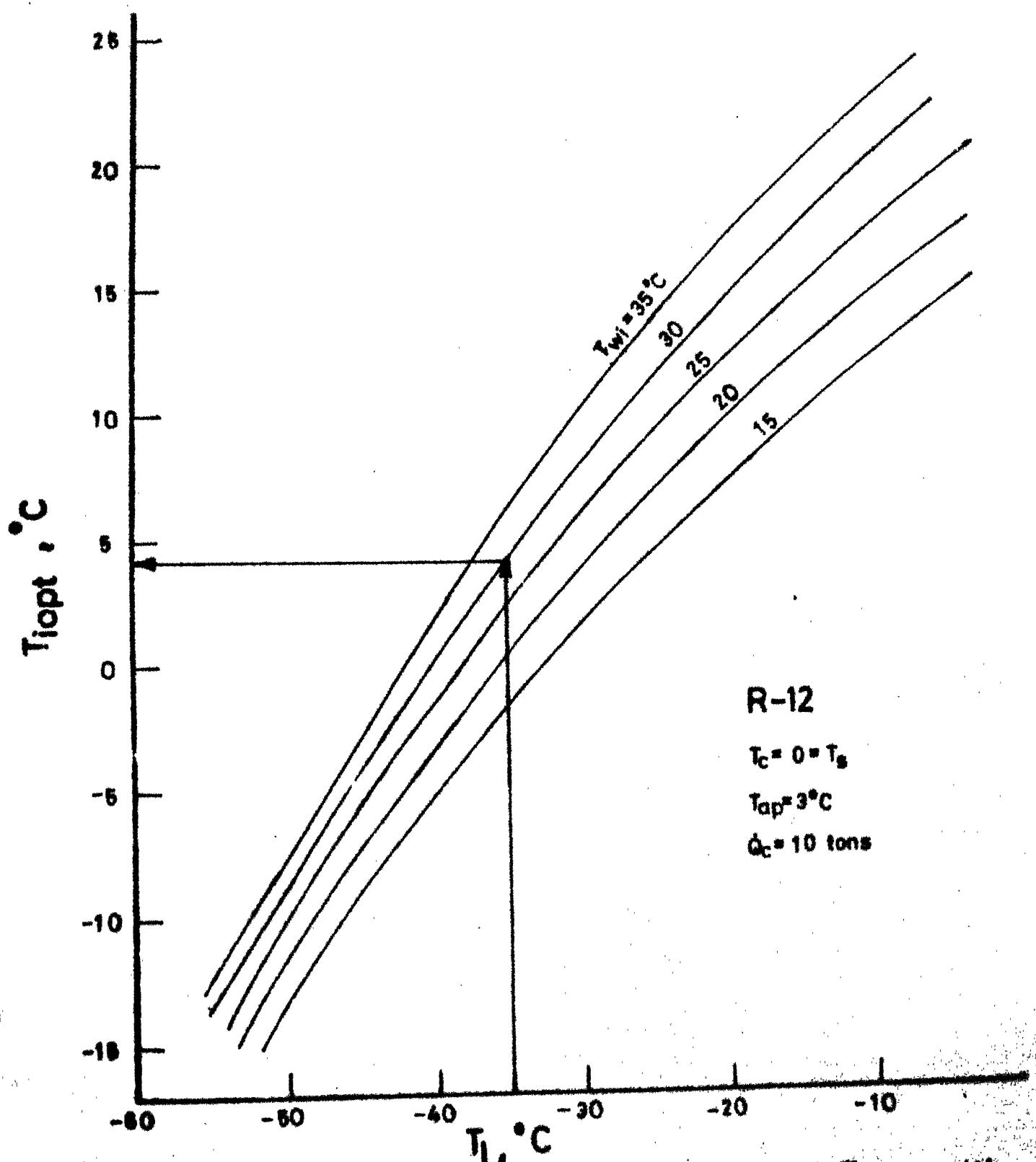
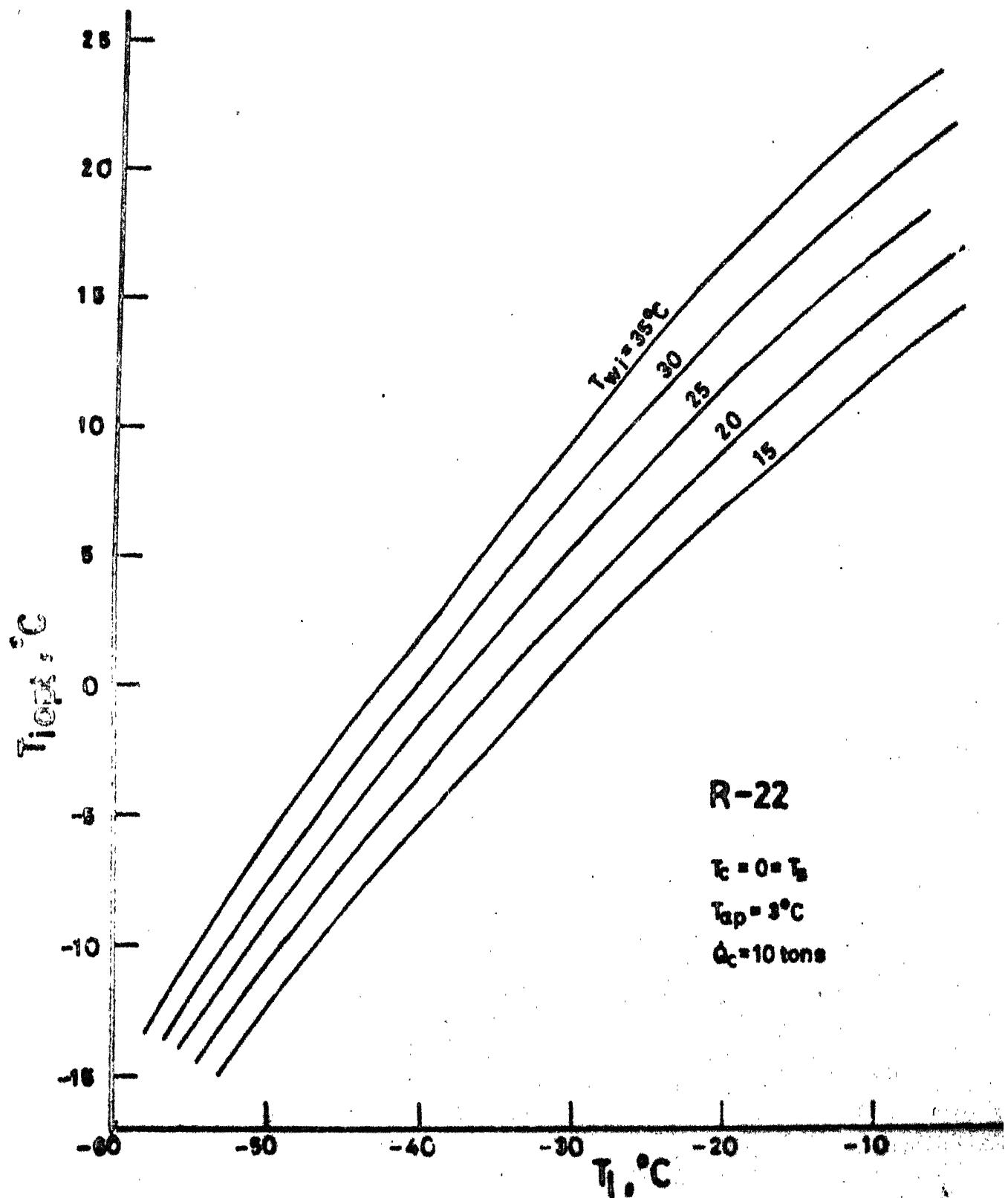
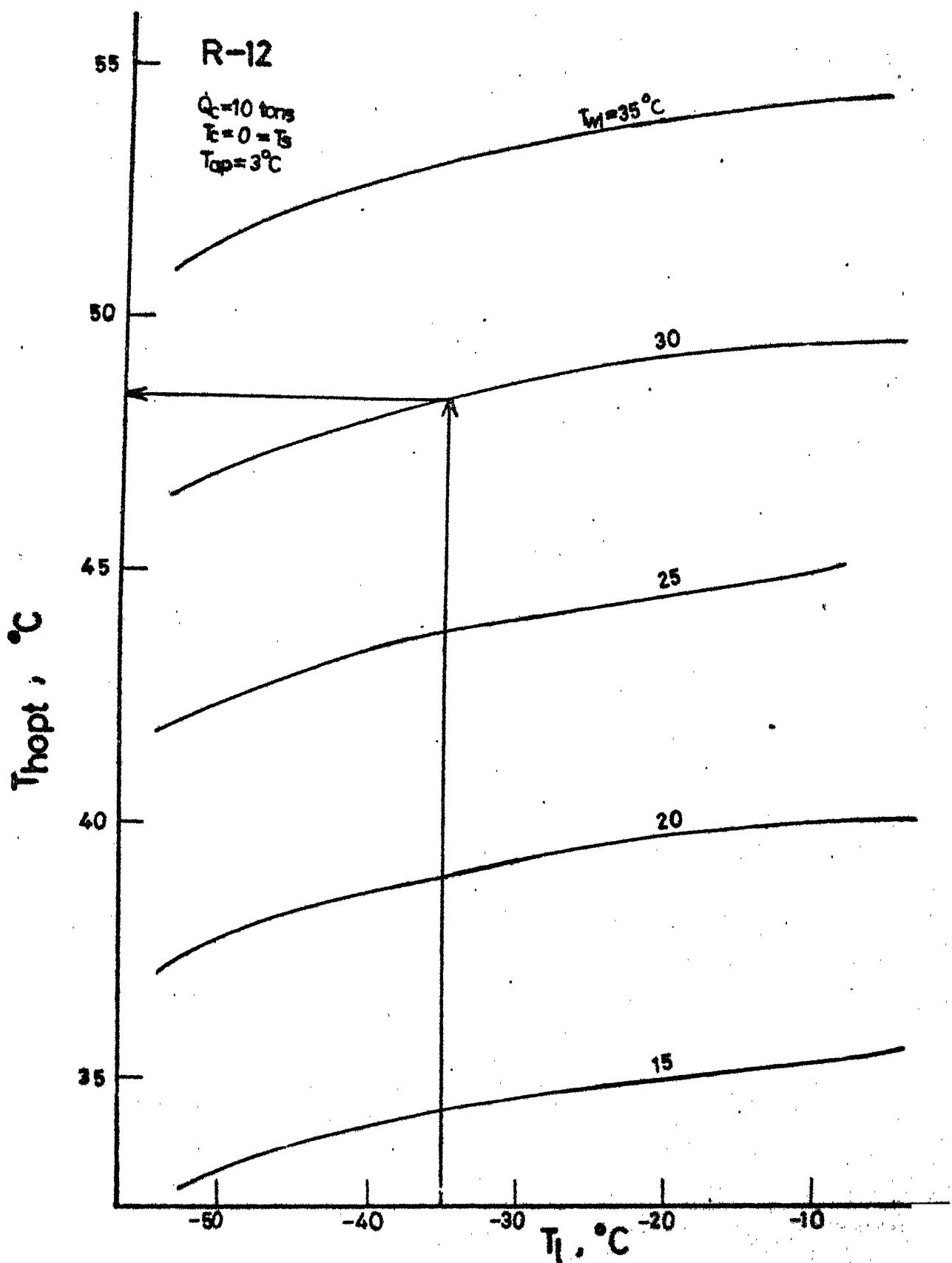


Fig 4.2a Variation of Optimal Interstage Temp with Evaporator and Ambient Temps



**Fig 4.2b Variation of Optimum Interstage Temp with Evaporator and Ambient Temp**



**Fig 4.3a Variation of Optimum Condensing Temp with Evaporator and Ambient Temps**

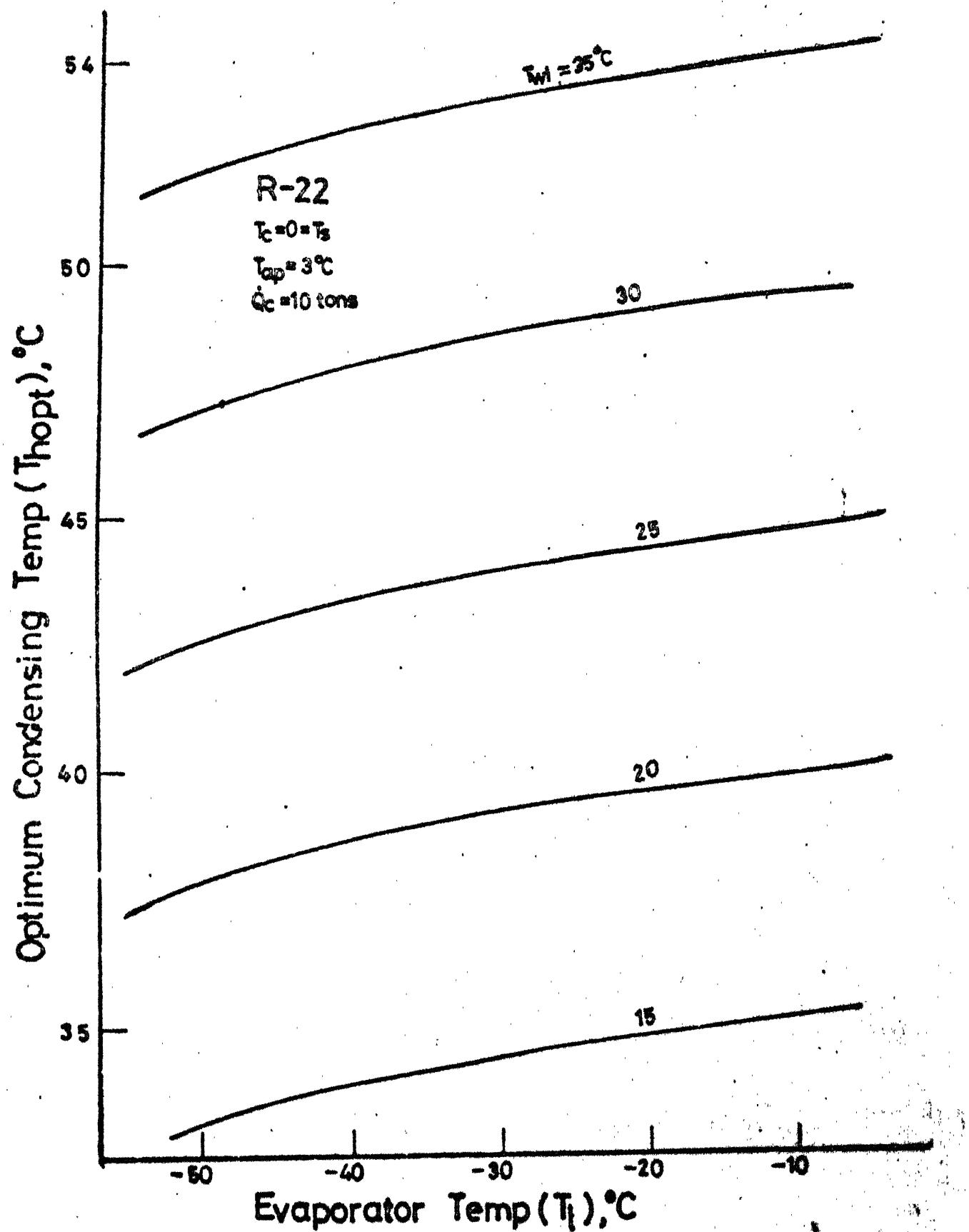


Fig 4.3b Variation of Optimum Condensing Temp with Evaporator and Ambient Temps

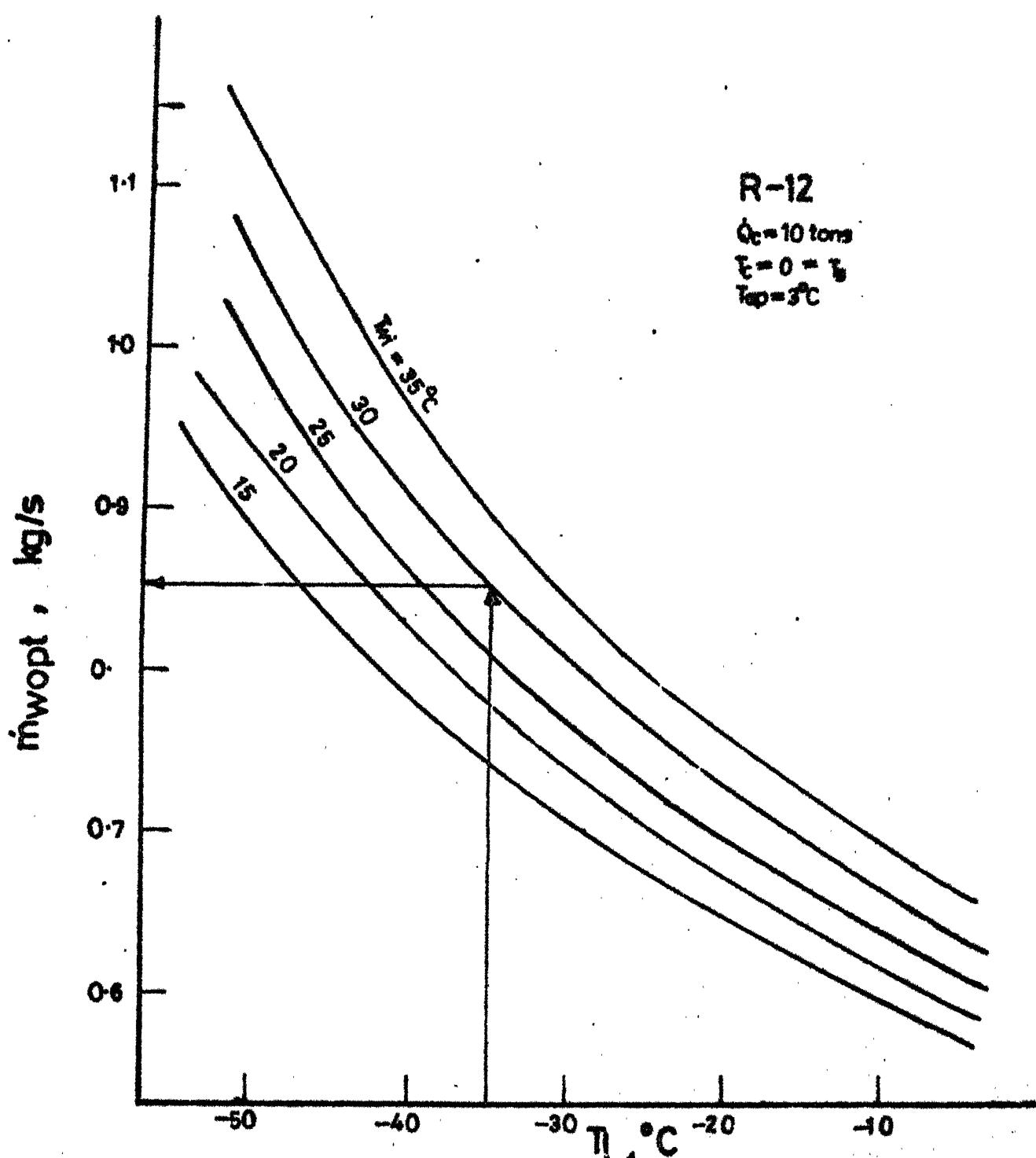


Fig 4.4a Variation of Optimum Cooling Water Rate with Evaporator and Ambient Temps

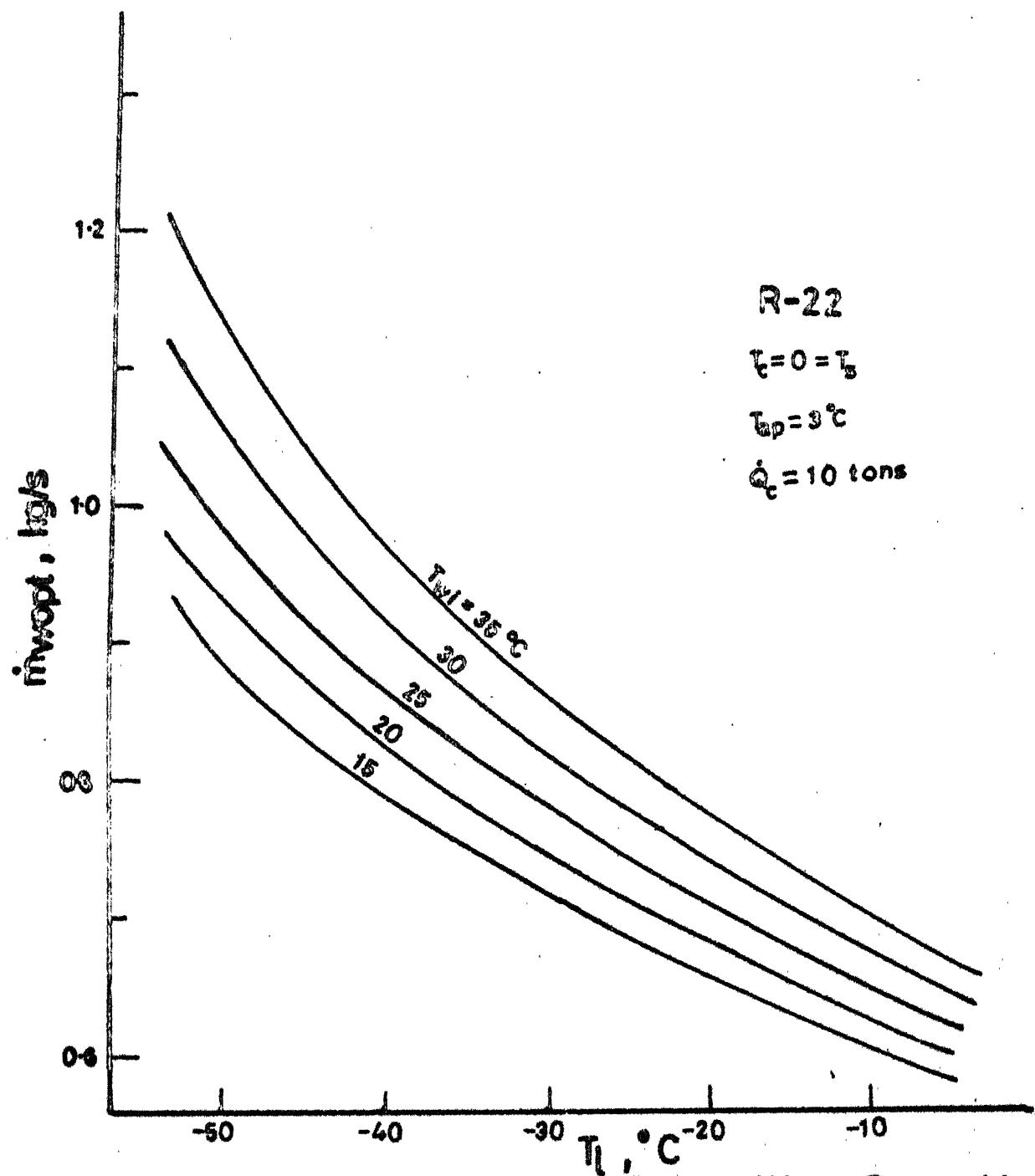


Fig 4.4b Variation of Optimum Cooling Water Rate with Evaporator and Ambient Temps

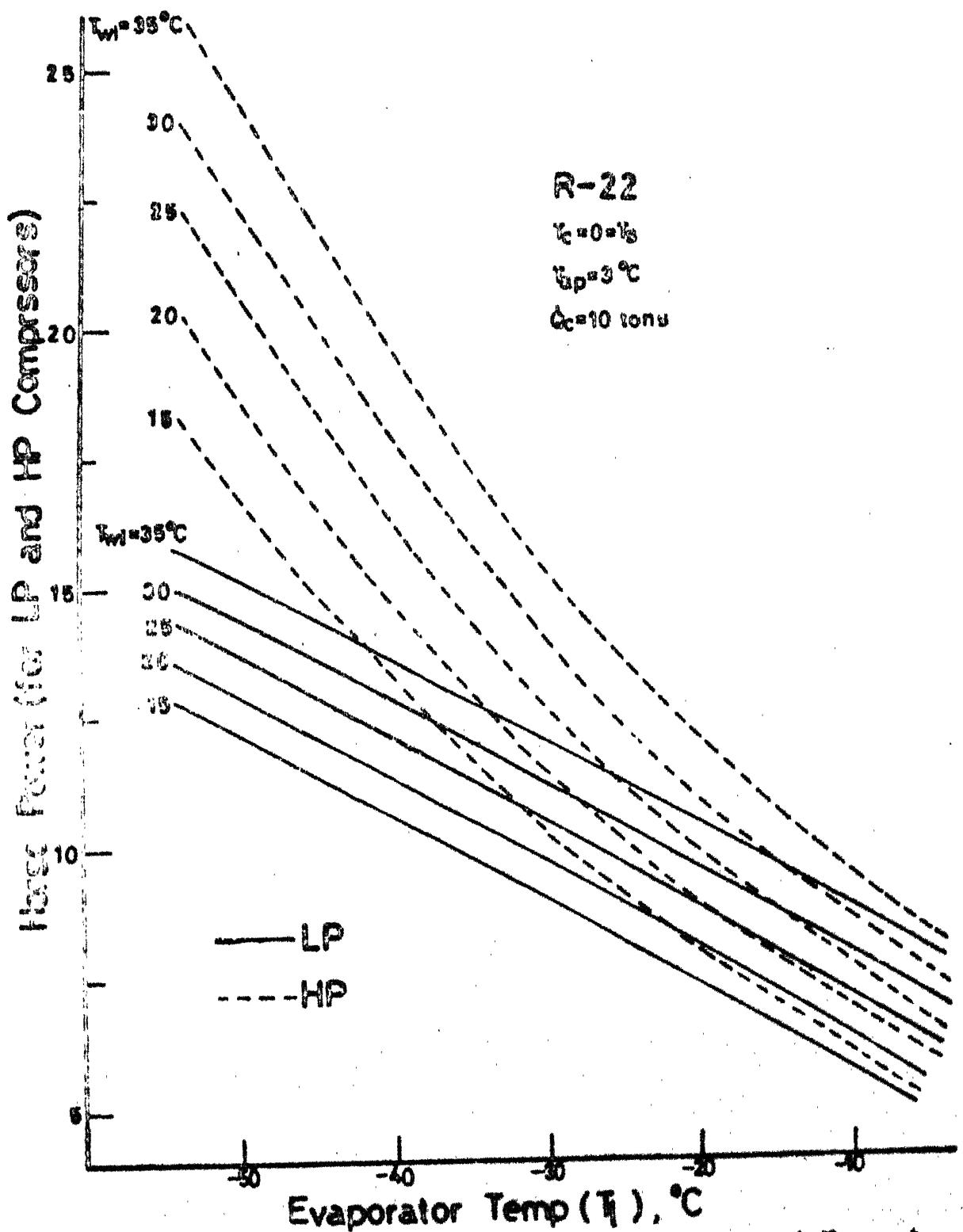


Fig 4.5b Variation of Horse Powers of LP and HP Compressors with Evaporator and Ambient Temperatures

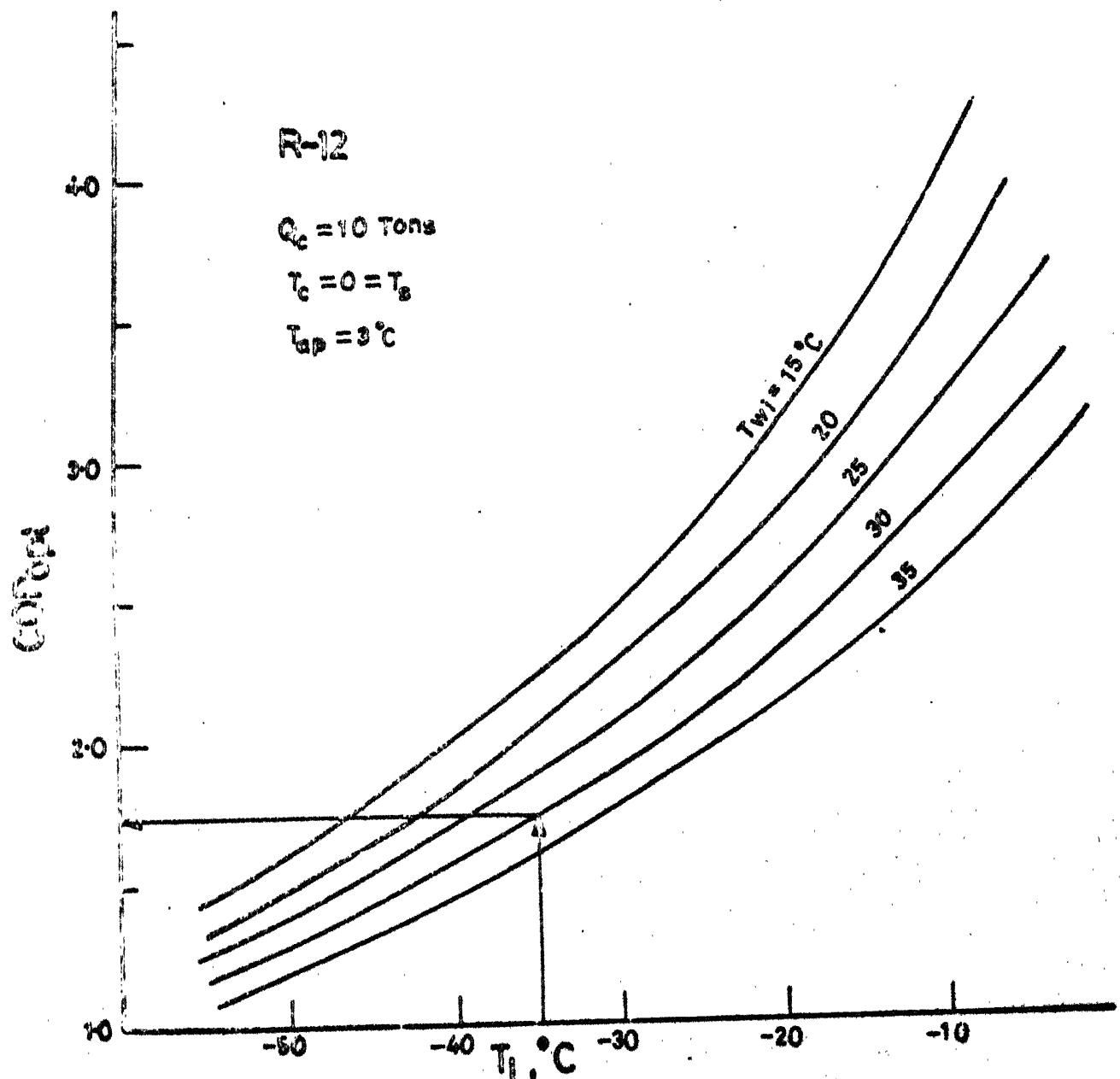


Fig 4.6a Variation of Optimum COP with Evaporator and Ambient Temps

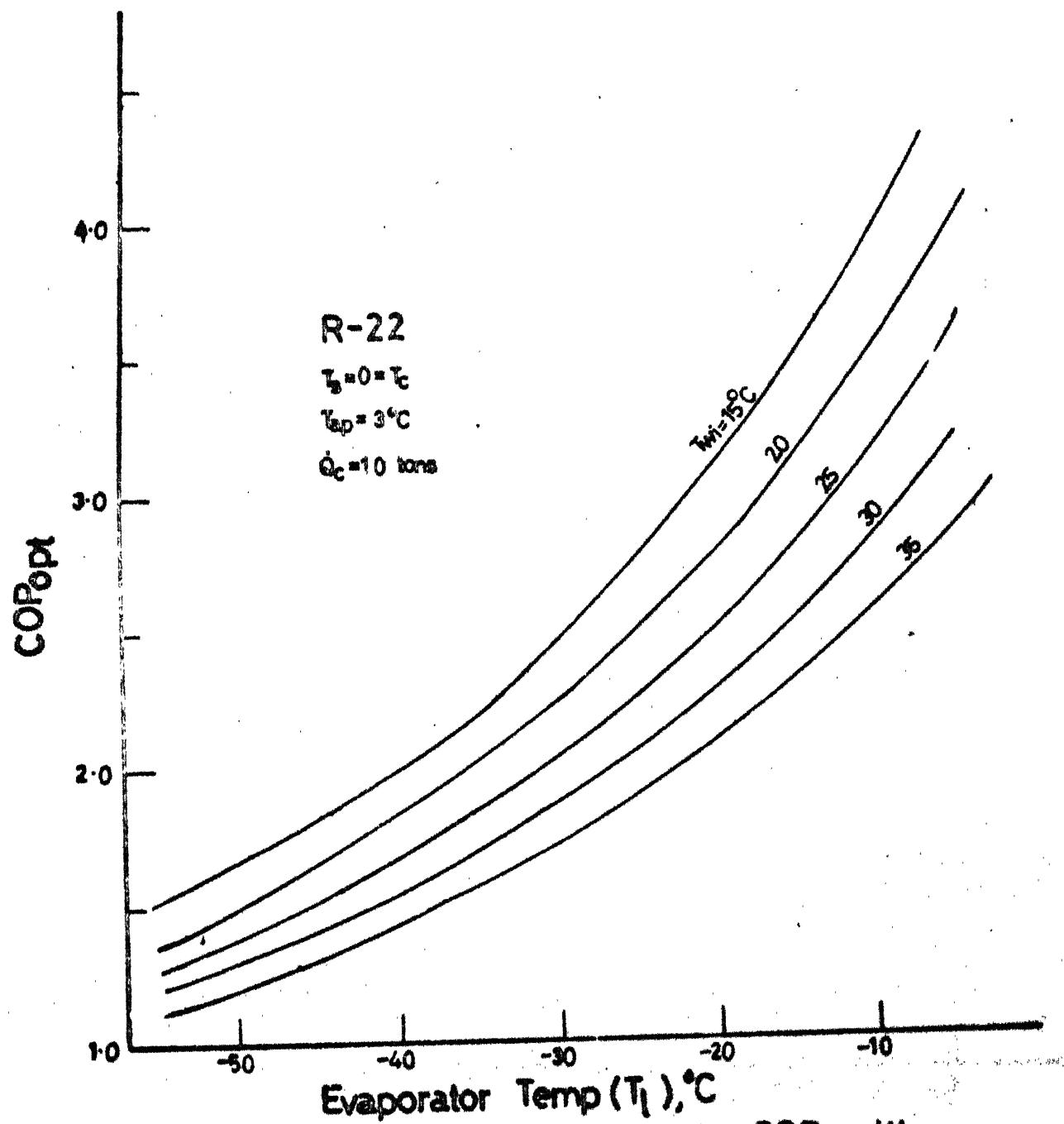


Fig 4.6b Variation of Optimum COP with Evaporator and Ambient Temps

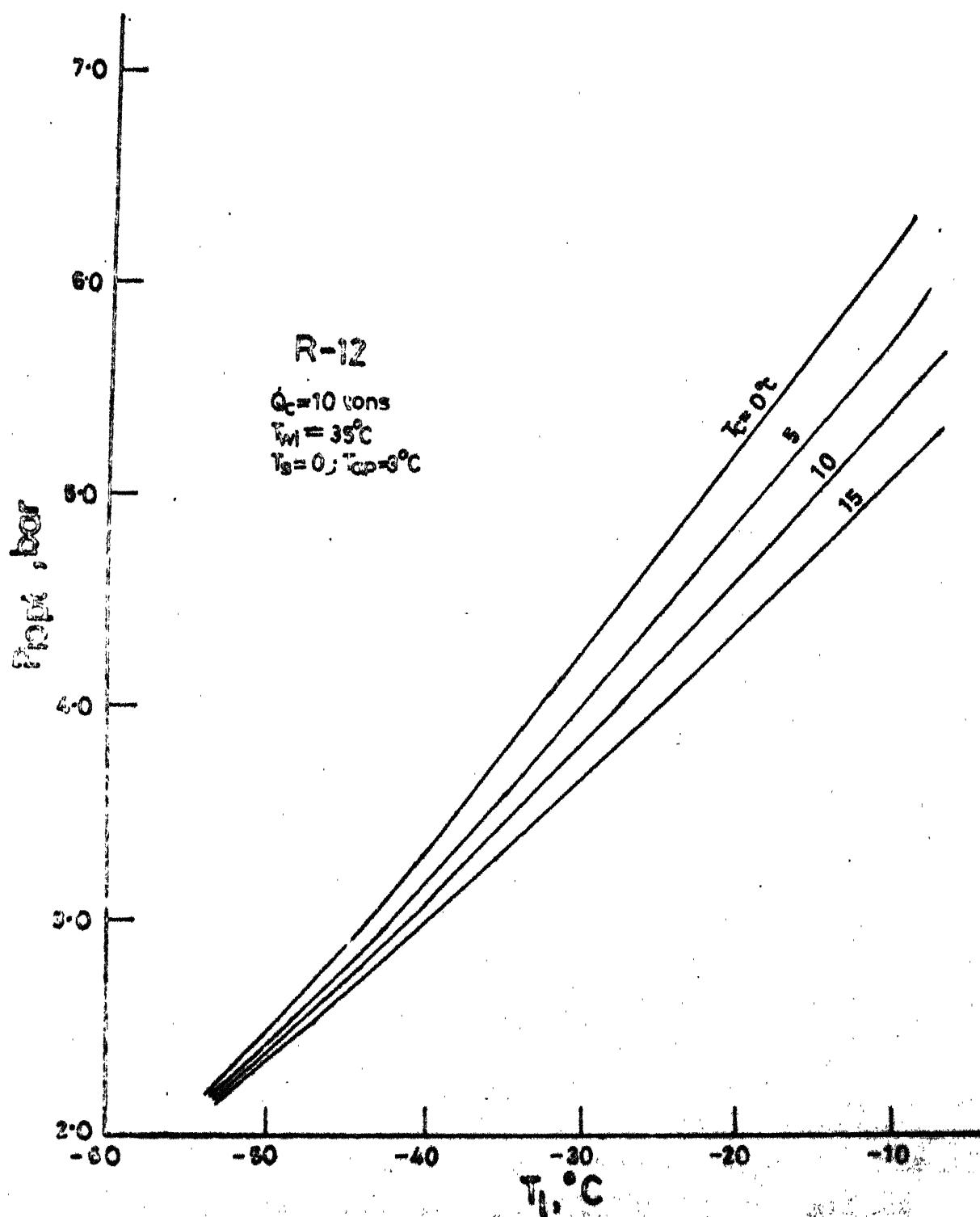


Fig 4.7a Effect of Subcooling on Optimum Interstage Pressure

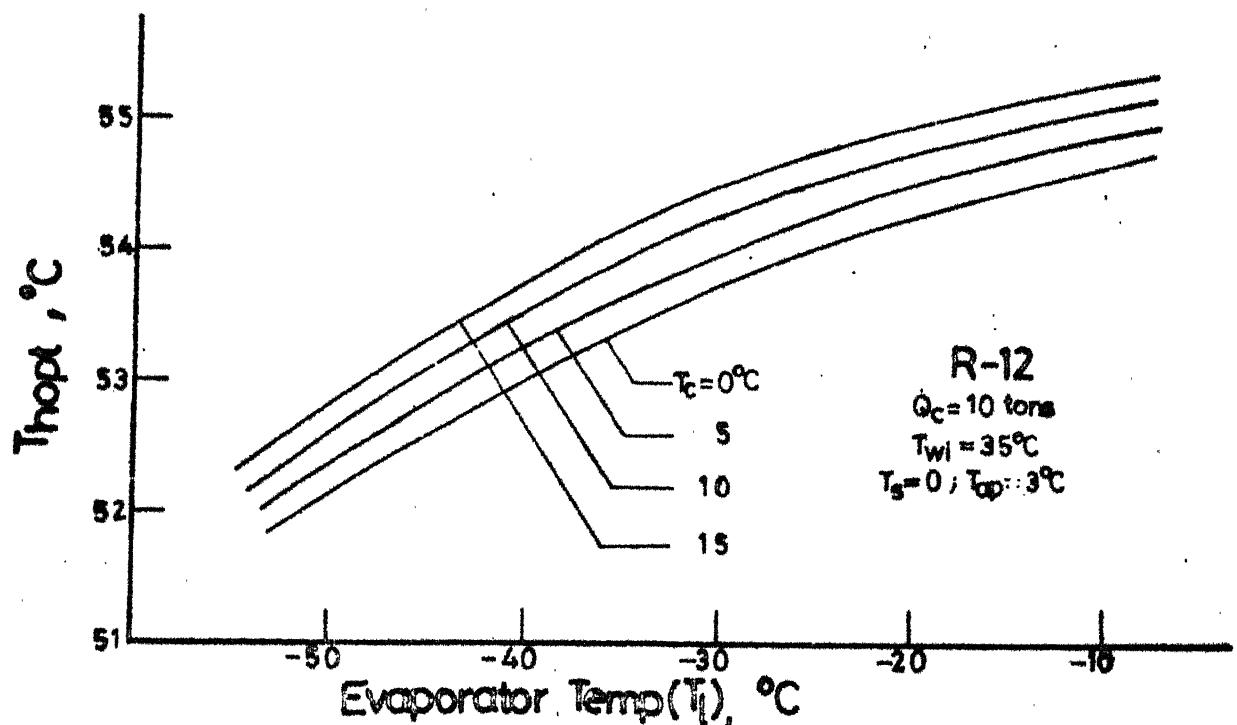


Fig 4.7b Effect of Subcooling on Optimum Condensing Temp

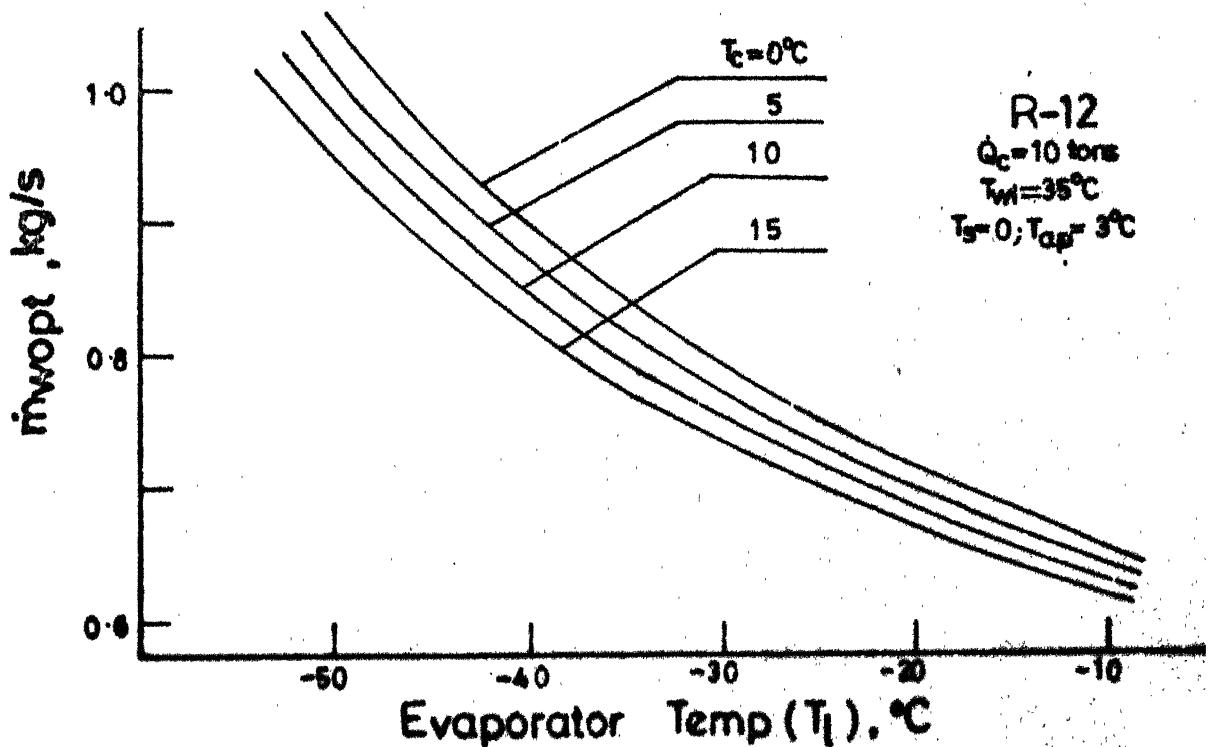


Fig 4.7c Effect of Subcooling on Optimum Cooling Water Rate

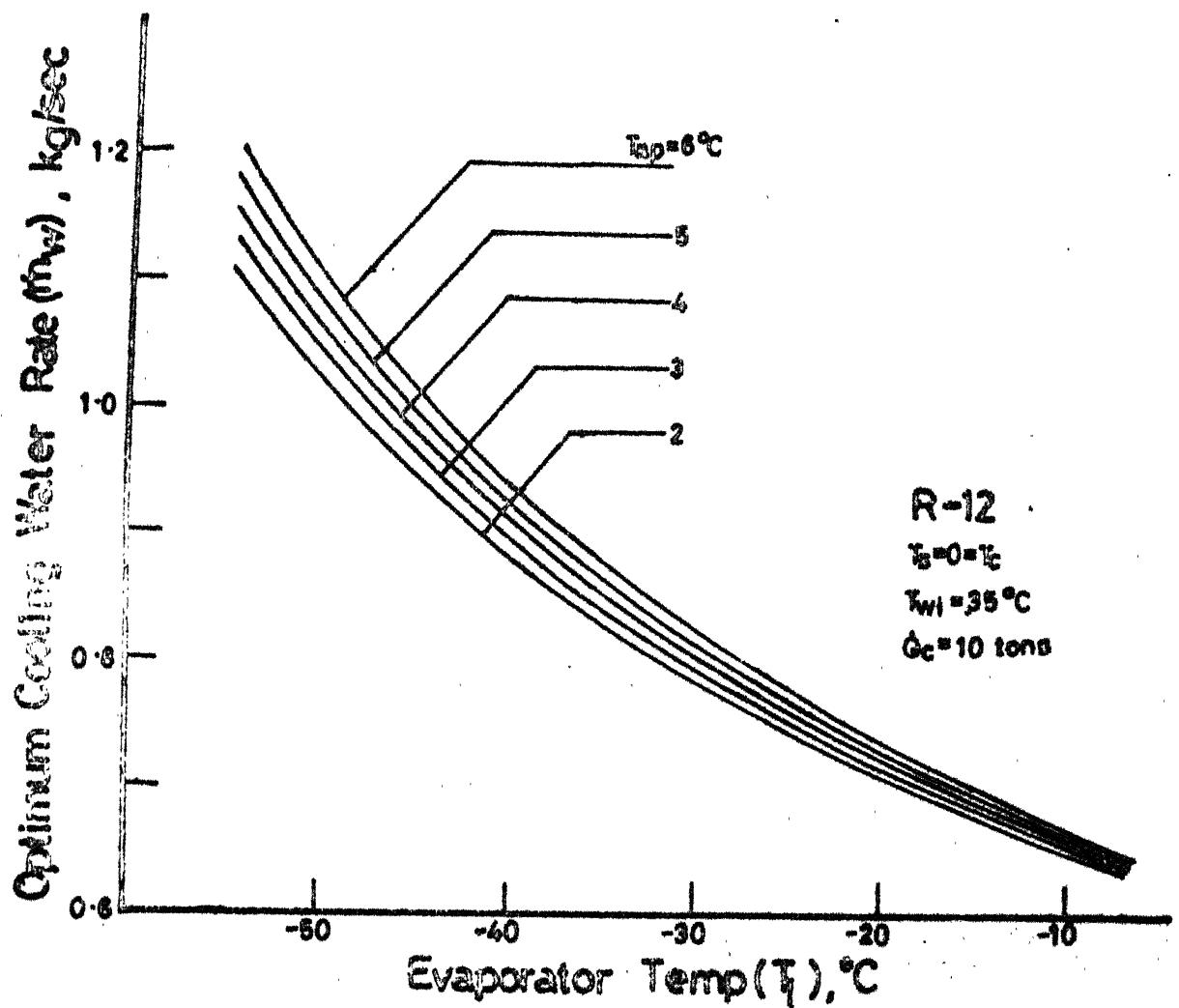


Fig 4.3 Effect of Approach Temperature ( $T_{ap}$ ) on Optimum Cooling Water Rate

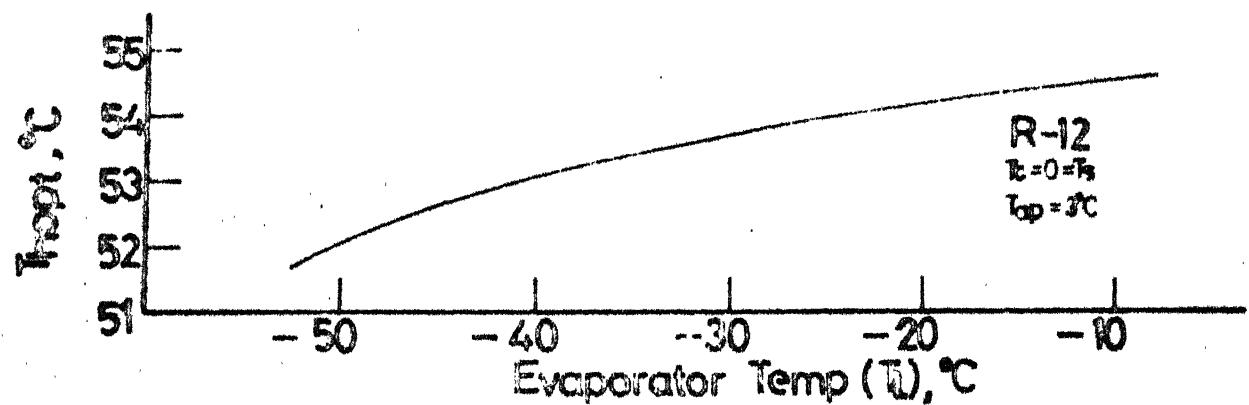


Fig 4.9a Optimum Condensing Temp with Variation in Ambient Temp

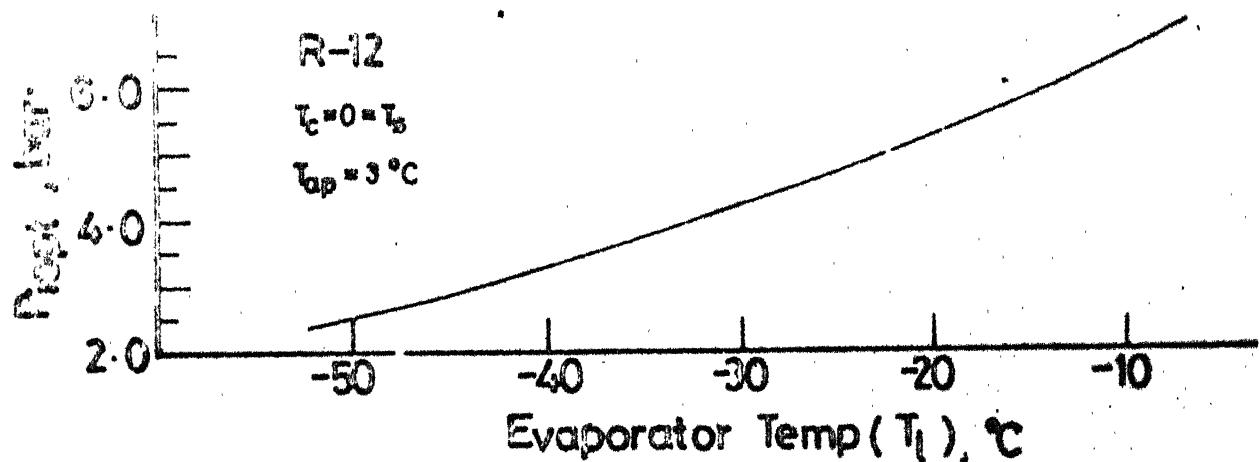


Fig 4.9b Optimum Interstage Pressure Including Effect of Variation in Ambient Temp

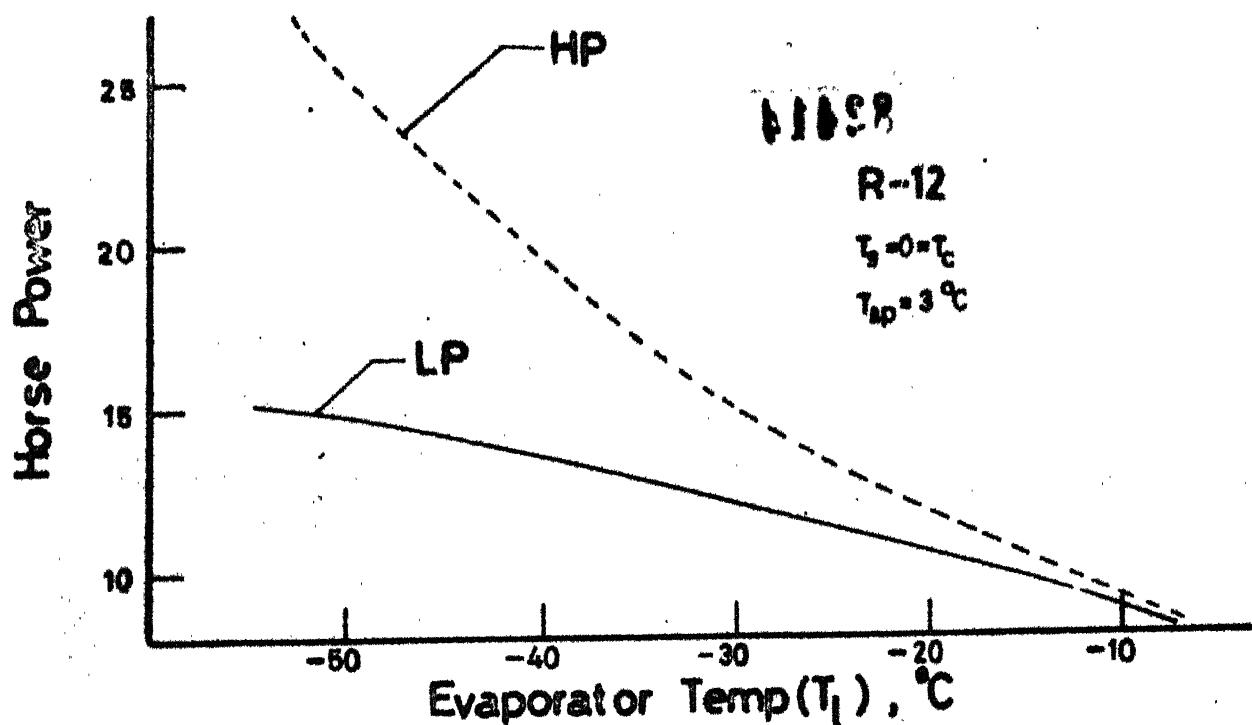


Fig 4.9c Horse Powers of LP and HP Compressors Including Effect of Variation in Ambient Temp

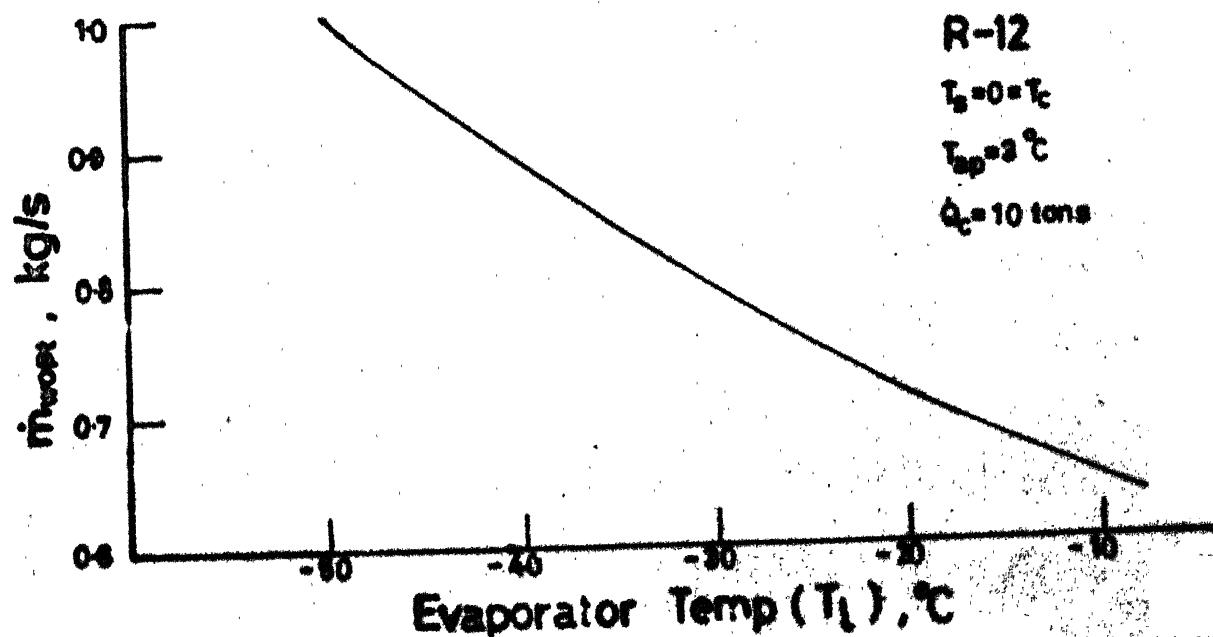


Fig 4.9d Optimal Cooling Water Rate Including Effect of Variation in Ambient Temp

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